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Problem #4

uniform probability distribution function on [0, 3]

So, the pdf is $f(x)=\frac{1}{3-0}=\frac{1}{3}$

(a) Mean

$$\mu=\int_0^3 x\ f(x)\,dx=\int_0^3 \frac{1}{3}x\,dx=\left[\frac{1}{3}\left(\frac{1}{2}\right)x^2\right]_0^3=\frac{1}{6}(3)^2-\frac{1}{6}(0)^2=\frac{9}{6}=\frac{3}{2}$$

(b) Standard deviation

$$Var(X)=\int_0^3 x^2\ f(x)\,dx-\mu^2$$

$$\int_0^3 x^2\ \frac{1}{3}\,dx=\left[\frac{1}{3}\left(\frac{1}{3}\right)x^3\right]_0^3=\frac{1}{9}(3)^3-\frac{1}{9}(0)^3=3$$

$$Var(x)=3-\left(\frac{3}{2}\right)^2=3-\frac{9}{4}=\frac{3}{4}$$

$$\sigma=\sqrt{Var(x)}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

Problem #3

$f(x)=\frac{2x}{9}$, [0, 3]

$$P(0<X<2)=\int_0^2 f(x)\,dx=\int_0^2 \frac{2x}{9}\,dx=\int_0^2 \frac{2}{9}x\,dx=\left[\frac{2}{9}\left(\frac{1}{2}\right)x^2\right]_0^2=\frac{1}{9}(2)^2-\frac{1}{9}(0)^2=\frac{4}{9}$$

Chapter 6, Page 484, Problem #7

$$\int\frac{t^2+1}{\sqrt{t}}\,dt=\int\left(\frac{t^2}{\sqrt{t}}+\frac{1}{\sqrt{t}}\right)dt=\int\left(t^{3/2}+t^{-1/2}\right)dt=\frac{2}{5}t^{5/2}+2t^{1/2}+C$$

Problem #9

$$\int\frac{e^{2x}+e^{-2x}}{e^{3x}}\,dx=\int\left[\frac{e^{2x}}{e^{3x}}+\frac{e^{-2x}}{e^{3x}}\right]dx=\int\left(e^{-x}+e^{-5x}\right)dx=-e^{-x}-\frac{1}{5}e^{-5x}+C$$