April 30, 2007

With an integral problem, how do you know if it is best by substitution or by parts?
First, substitution is the easiest; so I think about it first.

If I see $e^{\text{something}}$, try u = something

$(\text{something})^{\text{power}}$, try u = something

$\frac{1}{\textit{something}}$, try u = something

For integration by parts,

If I see $x^n \ln x$, let u = ln x and dv = x^n dx

$x^n e^{ax}$, let u = x^n and dv = e^{ax} dx

$x^n f(x)$, let u = x^n and dv = f(x) dx provided that I can integrate f(x)

Page 484, Problem #9

$$\int \frac{e^{2x}+e^{-2x}}{e^{3x}}\,dx = \int\left(\frac{e^{2x}}{e^{3x}}+\frac{e^{-2x}}{e^{3x}}\right)dx = \int\left(e^{-x}+e^{-5x}\right)dx = -e^{-x}-\frac{1}{5}e^{-5x}+C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}+C$$

Page 484, Problem #17

$$\int \frac{e^{x}}{e^{x}+1}\,dx = \int \frac{1}{e^{x}+1}e^{x}\,dx = \int \frac{1}{u}\,du = \ln u + C = \ln\left(e^{x}+1\right)+C$$

$$u = e^{x}+1$$

$$du = e^{x}\,dx$$

Page 493, Problem #3

$$\int_1^{e^2} x \ln x \, dx$$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$

$$\int u\,dv = uv - \int v\,du$$

$$\int_1^{e^2} x \ln x \, dx = \left[\ln x\left(\frac{1}{2}x^2\right)\right]_1^{e^2} - \int_1^{e^2} \frac{1}{2}x^2\left(\frac{1}{x}\right)dx = \left[\left(\frac{1}{2}x^2\right)\ln x\right]_1^{e^2} - \int_1^{e^2} \frac{1}{2}x\,dx$$

$$= \left[\left(\frac{1}{2}x^2\right)\ln x\right]_1^{e^2} - \left[\frac{1}{2}\left(\frac{1}{2}x^2\right)\right]_1^{e^2}$$

$$= \left[\left(\frac{1}{2}(e^2)^2\right)\ln e^2\right] - \left[\left(\frac{1}{2}(1)^2\right)\ln(1)\right] - \left(\left[\frac{1}{2}\left(\frac{1}{2}(e^2)^2\right)\right] - \left[\frac{1}{2}\left(\frac{1}{2}(1)^2\right)\right]\right)$$

$$= \quad \frac{1}{2}e^4(2) \quad - \quad \frac{1}{2}(0) \quad - \quad \frac{1}{4}e^4 \quad + \quad \frac{1}{4}$$

$$= \frac{3}{4}e^4 + \frac{1}{4}$$

$$\ln e^a = a$$