April 4, 2007
Answer questions Section 9.3

Problem #9

$$y = \sin^3 x \cos^4 x = (\sin x)^3 (\cos x)^4$$

$$y' = (\sin x)^3 \left[4(\cos x)^3 (-\sin x) \right] + (\cos x)^4 \left[3(\sin x)^2 (\cos x) \right]$$

$$= -4\sin^4 x \cos^3 x + 3\sin^2 x \cos^5 x$$

Problem #15

Find the equation of the line tangent to the curve y = sin 3x at x = $\frac{\pi}{12}$

Need:

1. Need a point

$$x = \frac{\pi}{12}$$

$$y = \sin\left[3\left(\frac{\pi}{12} \right) \right] = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

2. Need slope

$$m = \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{12}} = \left[3\cos 3x \right]_{x=\frac{\pi}{12}} = 3\cos\left[3\left(\frac{\pi}{12} \right) \right] = 3\cos\frac{\pi}{4} = 3\left(\frac{\sqrt{2}}{2} \right)$$

Equation

$$y - \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}\left(x - \frac{\pi}{12} \right)$$

Problem #17

Find max and min of y = sin x + cos x for x in the interval [0, 2π]

Need critical point(s); solve y' = 0

$$\frac{dy}{dx} = \cos x - \sin x$$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Max or min?

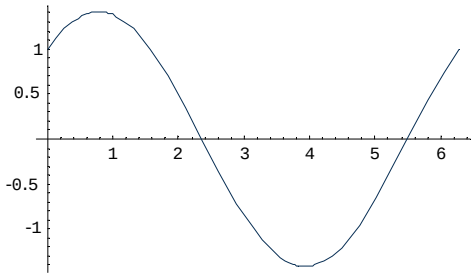
$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$x = \frac{\pi}{4}, \frac{d^2y}{dx^2} = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

max

$$x = \frac{5\pi}{4}, \frac{d^2y}{dx^2} = -\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4} = -\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right) = +\sqrt{2}$$

min



Problem #19

max and min of

$$f(x) = x + \sin x, \quad [0, 2\pi]$$

$$f'(x) = 1 + \cos x$$

$f'(x) = 0$ when $1 + \cos x = 0$

$$\cos x = -1$$

$$x = \pi$$

$$f''(x) = 0 - \sin x$$

$$f''(\pi) = -\sin\pi = 0$$

$$f'(\frac{5\pi}{6}) = 1 + \cos\frac{5\pi}{6} = 1 - \frac{\sqrt{3}}{2} > 0$$

$$f'(\frac{7\pi}{6}) = 1 + \cos\frac{7\pi}{6} = 1 - \frac{\sqrt{3}}{2} > 0$$

inflection point