April 6, 2007

Questions from Section 9.4
Problem #13

$$\int x^2 \sin x \, dx$$

$$u = x^2 \qquad dv = \sin x \, dx$$

$$du = 2x \, dx \qquad v = -\cos x$$

$$\int x^2 \sin x \, dx = x^2(-\cos x) - \int(-\cos x) 2x \, dx$$

$$= -x^2 \cos x + \int 2x \cos x \, dx$$

$$u = 2x \qquad dv = \cos x \, dx$$

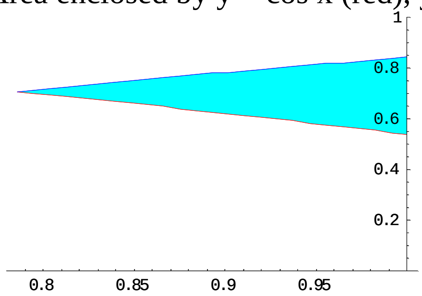
$$du = 2 \, dx \qquad v = \sin x$$

$$-x^2 \cos x + \int 2x \cos x \, dx = -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx$$

$$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

Problem #17
Area enclosed by y = cos x (red), y = sin x (blue), x = 1, x = π/4



$$\int_{\pi/4}^{1} (\sin x - \cos x) \, dx = [-\cos x - \sin x]_{\pi/4}^{1} = -\cos 1 - \sin 1 - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$$

$$= -\cos 1 - \sin 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = -\cos 1 - \sin 1 + \sqrt{2} \approx 0.032$$

Section 8.5 Problem #10
We want the change in z as we move from the point (1, 2) to the point (1.02, 1.5)
$dz = f_x(a, b)\, dx + f_y(a, b)\, dy$

$$f(x, y) = \ln(x^2 + y^2)$$

$$(a, b) = (1, 2), \Delta x = 0.02, \Delta y = -0.5$$

$$f_x = \frac{2x}{x^2 + y^2}; \quad f_y = \frac{2y}{x^2 + y^2}$$

$$dz = \frac{1(2)}{1^2 + 2^2}(0.02) + \frac{2(2)}{1^2 + 2^2}(-0.5) = \frac{2}{5}(0.02) - \frac{4}{5}(0.5) = 0.008 - 0.4 = -0.392$$

Section 8.6, Problem #12

$$\iint_D 6xy^2 \, dA; \;\; D = \{(x, y) : 1 \le x \le 2, 2 \le y \le 3\}$$

$$\int_2^3 \left[\int_1^2 6xy^2 \, dx \right] dy$$

$$\int_1^2 6xy^2 \, dx = \int_1^2 6y^2 x \, dx = \left[6y^2 \left(\frac{1}{2} \right) x^2 \right]_1^2 = \left[3y^2 x^2 \right]_1^2 = 3y^2 (2)^2 - 3y^2 (1)^2$$

$$= 12y^2 - 3y^2 = 9y^2$$

$$\int_2^3 9y^2 \, dy = \left[9 \left(\frac{1}{3} \right) y^3 \right]_2^3 = 3(3)^3 - 3(2^3) = 81 - 24 = 57$$