February 16, 2007
Section 7.1

Problem #3

$$\int_1^{e^2} x \ln x \, dx$$

$$u = \ln x \qquad dv = x\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$$

$$\int_1^{e^2} x \ln x \, dx = \left[\ln x\left(\frac{1}{2}x^2\right)\right]_1^{e^2} - \int_1^{e^2} \frac{1}{2}x^2\left(\frac{1}{x}\right)dx = \left[\frac{1}{2}x^2 \ln x\right]_1^{e^2} - \frac{1}{2}\int_1^{e^2} x\,dx$$

$$= \left[\frac{1}{2}x^2 \ln x\right]_1^{e^2} - \frac{1}{2}\left[\frac{1}{2}x^2\right]_1^{e^2} = \left[\frac{1}{2}\left(e^2\right)^2 \ln e^2\right] - \left[\frac{1}{2}1^2 \ln 1\right] - \left\{\frac{1}{2}\left[\frac{1}{2}(e^2)^2\right] - \frac{1}{2}\left[\frac{1}{2}1^2\right]\right\}$$

$$= \frac{1}{2}e^4(2) - \frac{1}{2}(0) - \frac{1}{4}e^4 + \frac{1}{4} = \frac{3}{4}e^4 + \frac{1}{4}$$

Problem #13

$$\int x^5\sqrt{1+x^3}\,dx$$

$$u = x^3 \qquad dv = x^2\sqrt{1+x^3}\,dx$$

$$du = 3x^2\,dx \qquad v = \frac{2}{9}\left(1+x^3\right)^{3/2}$$

$$\int x^5\sqrt{1+x^3}\,dx = x^3\left[\frac{2}{9}\left(1+x^3\right)^{3/2}\right] - \frac{2}{9}(3)\int x^2\left(1+x^3\right)^{3/2}dx$$

$$= \frac{2}{9}x^3\left(1+x^3\right)^{3/2} - \frac{2}{3}\int x^2\left(1+x^3\right)^{3/2}dx$$

$$= \frac{2}{9}x^3\left(1+x^3\right)^{3/2} - \frac{2}{3}\left[\frac{1}{3}\left(\frac{2}{5}\right)\left(1+x^3\right)^{5/2}\right] + C$$

$$= \frac{2}{9}x^3\left(1+x^3\right)^{3/2} - \frac{4}{45}\left(1+x^3\right)^{5/2} + C$$

Side Work:

$$\int x^2\sqrt{1+x^3}\,dx = \int x^2\left(1+x^3\right)^{1/2}dx = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\left(\frac{2}{3}\right)u^{3/2} + C = \frac{2}{9}\left(1+x^3\right)^{3/2} + C$$

$$u = 1 + x^3$$

$$du = 3x^2\,dx$$

$$\frac{1}{3}du = x^2\,dx$$

Problem #15

$$\int x^3\left(1+x^2\right)^{10}\ dx$$

$$u = x^2 \qquad dv = x\left(1+x^2\right)^{10}\ dx$$

$$du = 2x\ dx \qquad v = \frac{1}{22}\left(1+x^2\right)^{11}$$

$$\int x^3\left(1+x^2\right)^{10}\ dx = x^2\left[\frac{1}{22}\left(1+x^2\right)^{11}\right] - \int \frac{1}{22}\left(1+x^2\right)^{11}(2x)\ dx$$

$$= \frac{1}{22}x^2\left(1+x^2\right)^{10} - \frac{1}{22}\int\left(1+x^2\right)^{11}(2x)\ dx$$

$$= \frac{1}{22}x^2\left(1+x^2\right)^{10} - \frac{1}{22}\left(\frac{1}{12}\right)\left(1+x^2\right)^{12} + C$$

$$= \frac{1}{22}x^2\left(1+x^2\right)^{10} - \frac{1}{264}\left(1+x^2\right)^{12} + C$$

$$v = \int x\left(1+x^2\right)^{10}\ dx = \frac{1}{2}\int u^{10}\ du = \frac{1}{2}\left(\frac{1}{11}\right)u^{11} + C = \frac{1}{22}\left(1+x^2\right)^{11} + C$$

$$u = 1+x^2$$

$$du = 2x\ dx$$

$$\frac{1}{2}\ du = x\ dx$$

$$\int\left(1+x^2\right)^{11}(2x)\ dx = \int u^{11}\ du = \frac{1}{12}u^{12} + C = \frac{1}{12}\left(1+x^2\right)^{12} + C$$

Problem 1

$$\int 2x\ e^{2x}\ dx$$

$$u = 2x \qquad dv = e^{2x}\ dx$$

$$du = 2\ dx \qquad v = \frac{1}{2}e^{2x}$$

$$\int u\ dv = uv - \int v\ du$$

$$\int 2x\ e^{2x}\ dx = 2x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}(2)\ dx = xe^{2x} - \int e^{2x}\ dx$$

$$= xe^{2x} - \frac{1}{2}e^{2x} + C$$