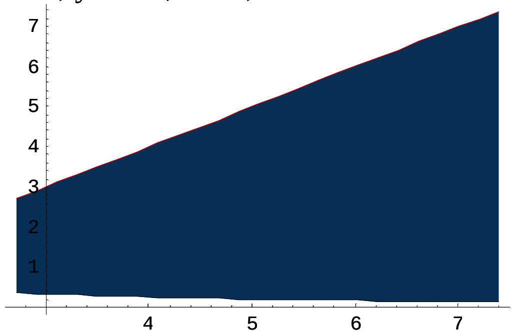
February 5
Answer questions
Section 6.6

Problem 37
y = x, y = 1/x, x = e, x = e^2



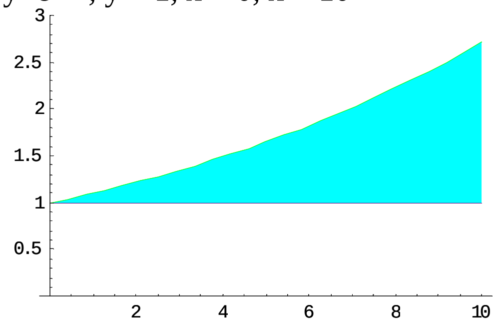
The line y = x is red

$$\int_e^{e^2}\left(x-\frac{1}{x}\right)dx=\left[\frac{1}{2}x^2-\ln x\right]_e^{e^2}=\left[\frac{1}{2}\left(e^2\right)^2-\ln e^2\right]-\left[\frac{1}{2}e^2-\ln e\right]$$

$$=\frac{1}{2}e^4-2-\frac{1}{2}e^2+1=\frac{1}{2}e^4-\frac{1}{2}e^2-1$$

Know: $\ln\left(e^a\right)=a\ln e=a(1)=a$

Problem #3
$y=e^{0.1x}$, y = 1, x = 0, x = 10



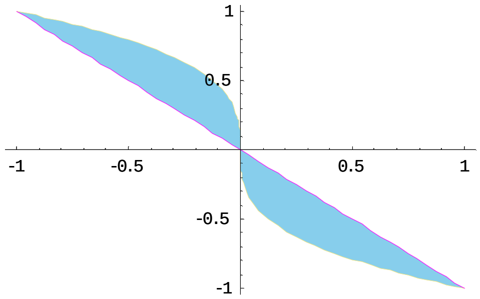
The line y = 1 is purple and the curve y = $e^{0.1x}$ is green

$$\int_0^{10}\left(e^{0.1x}-1\right)dx=\left[\frac{1}{0.1}e^{0.1x}-x\right]_0^{10}=\left[10e^{0.1(10)}-10\right]-\left[10e^{0.1(0)}-0\right]$$

$$=10e^1-10-10e^0+0=10e-20=10(e-2)$$

$$\frac{1}{0.1}=\frac{1}{\frac{1}{10}}=10$$

Problem #27
$y=-\sqrt[3]{x},\ y=-x,\ x=-1,\ x=1$
If x < 0, $x^{1/3} < 0$; So $-x^{1/3} > 0$



So I have 2 congruent regions

$$2\int_0^1 \left(-x-\left(-\sqrt[3]{x}\right)\right)dx = 2\int_0^1 \left(-x+x^{1/3}\right)dx = 2\left[-\frac{1}{2}x^2+\frac{3}{4}x^{4/3}\right]_0^1$$

$$=2\left[-\frac{1}{2}1^2+\frac{3}{4}1^{4/3}\right]-2\left[-\frac{1}{2}0^2+\frac{3}{4}0^{4/3}\right]=2\left[-\frac{1}{2}+\frac{3}{4}\right]=2\left(\frac{1}{4}\right)=\frac{1}{2}$$

Problem #9

$y = x^2 - 2x + 1$, $y = x + 1$

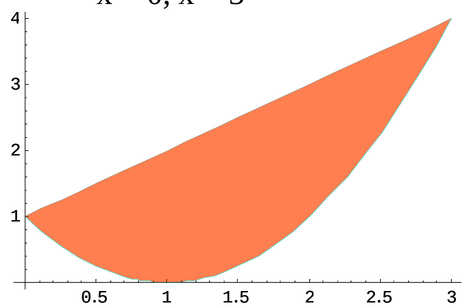
Where do these intersect?

$x^2 - 2x + 1 = x + 1$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$, $x = 3$



$$\int_0^3 \left[(x+1)-\left(x^2-2x+1\right)\right]dx = \int_0^3 \left(3x-x^2\right)dx = \left[\frac{3}{2}x^2-\frac{1}{3}x^3\right]_0^3$$

$$=\left[\frac{3}{2}3^2-\frac{1}{3}3^3\right]-\left[\frac{3}{2}0^2-\frac{1}{3}0^3\right]=\frac{27}{2}-9-0=\frac{9}{2}$$