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Answer Questions Section 8.3

Problem #13
$f(x, y) = x^3 + y^3 - 3xy - 1$
Find critical points
$f_x = 3x^2 - 3y = 0 \rightarrow x^2 = y$
$f_y = 3y^2 - 3x = 0 \rightarrow y^2 = x$
$(y^2)^2 = y$
$y^4 - y = 0$
$y(y^3 - 1) = 0$
$y = 0, y = 1$
Critical points (0, 0) and (1, 1)
Determine whether relative max, relative min, or saddle point
First need $D(x, y) = f_{xx}f_{yy} - (f_{xy})^2$
$f_x = 3x^2 - 3y$
$f_{xx} = 6x$
$f_{xy} = -3$
$f_y = 3y^2 - 3x$
$f_{yy} = 6y$
So, $D(x, y) = (6x)(6y) - (-3)^2 = 36xy - 9$
Now, for the point (0, 0), $D(0, 0) = -9$ and we have a saddle point
For the point (1, 1), $D(1, 1) = 36 - 9 = 27$; then we look at $f_{xx}(1, 1) = 6(1) = 6$ and we have a relative minimum

Problem #15
$f(x, y) = xy - x^3 - y^2$

$f_x = y - 3x^2$; $f_{xx} = -6x$; $f_{xy} = 1$
$f_y = x - 2y$; $f_{yy} = -2$

Solve: $y - 3x^2 = 0$ or $y = 3x^2$
$x - 2y = 0$ or $x = 2y$
So $x = 2(3x^2)$ or $x - 6x^2 = 0$ or $x(1 - 6x) = 0$
$x = 0, x = 1/6$
Points (0, 0) and (1/6, 1/12)

$D(x, y) = f_{xx}f_{yy} - (f_{xy})^2 = (-6x)(-2) - (1)^2 = 12x - 1$

Point (0, 0), $D(0, 0) = -1$; saddle point
Point(1/6, 1/12), $D(1/6, 1/12) = 12(1/6) - 1 = 2 - 1 = 1$
$f_{xx} = -6(1/6) = -1$; relative max