

March 26, 2007

Section 8.5

Problem #3

$$f(x, y) = x^2 y^3, (a, b) = (3, 2), \Delta x = 0.1, \Delta y = -0.1$$

$$\Delta z = f(x + \Delta x, y + \Delta y) \approx f_x(a, b) \Delta x + f_y(a, b) \Delta y$$

$$f_x = 2xy^3; f_x(3, 2) = 2(3)(2^3) = 48$$

$$f_y = 3x^2 y^2; f_y(3, 2) = 3(3^2)(2^2) = 108$$

$$\Delta z \approx 48(0.1) + 108(-0.1) = 4.8 - 10.8 = -6$$

Problem #7

$$f(x, y) = \frac{x}{x^2 + y^2}, (a, b) = (3, 4); \Delta x = -0.1, \Delta y = 0.2$$

$$f_x = \frac{(x^2 + y^2)(1) - x(2x)}{(x^2 + y^2)^2}; f_x(3, 4) = \frac{3^2 + 4^2 - 2(3^2)}{(3^2 + 4^2)^2} = \frac{7}{625}$$

$$f_y = \frac{(x^2 + y^2)(0) - x(2y)}{(x^2 + y^2)^2} = \frac{-2xy}{(x^2 + y^2)^2}; f_y(3, 4) = \frac{-2(3)(4)}{(3^2 + 4^2)^2} = -\frac{24}{625}$$

$$\Delta z \approx \frac{7}{625}(-0.1) - \frac{24}{625}(0.2) = -8.6 \times 10^{-6}$$

Problem #13

$$f(x, y) = e^{x+y}; (a, b) = (1, -1), \Delta x = 0.02, \Delta y = 0.03$$

$$f_x = e^{x+y}(1); f_x(1, -1) = e^{1-1} e^0 = 1$$

$$f_y = e^{x+y}(1); f_y(1, -1) = 1$$

$$\Delta z \approx 1(0.02) + 1(0.03) = 0.05$$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y) = e^{1.02-0.97} - e^{1-1} = 0.0513$$