March 28, 2007
Section 8.6

Problem #13

$$\iint_D y\sqrt{y^2+x}\ dA;\ D=\{(x,y):0\le x\le 1, 0\le y\le 1\}$$

$$\int_0^1\left[\int_0^1 y\sqrt{y^2+x}\ dy\right]dx$$

$$\int_0^1 y\sqrt{y^2+x}\ dy$$

		y	u
$u=y^2+x$			
$du=2y\ dy$	UP	1	$1+x$
$\frac{1}{2}du=y\ dy$	LOW	0	$0+x$

$$\int_0^1 y\sqrt{y^2+x}\ dy=\frac{1}{2}\int_x^{1+x}\sqrt{u}\ du=\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_x^{1+x}=\frac{1}{3}(1+x)^{3/2}-\frac{1}{3}x^{3/2}$$

$$\int_0^1\left[\int_0^1 y\sqrt{y^2+x}\ dy\right]dx=\int_0^1\left[\frac{1}{3}(1+x)^{3/2}-\frac{1}{3}x^{3/2}\right]dx$$

$$=\frac{1}{3}\left[\frac{2}{5}(1+x)^{5/2}-\frac{2}{5}x^{5/2}\right]_0^1=\frac{2}{15}\left[\left[(1+1)^{5/2}-1^{5/2}\right]-\left[(1+0)^{5/2}-0^{5/2}\right]\right]$$

$$=\frac{2}{15}\left[2^{5/2}-1-1+0\right]=\frac{2}{15}\left[2^{5/2}-2\right]$$

Problem #17

$$\iint_D e^{x+y}\ dA;\ \ D=\{(x,y):0\le x\le 2, 0\le y\le 3\}$$

$$\int_0^3\left[\int_0^2 e^xe^y\ dx\right]dy$$

$$\int_0^2 e^xe^y\ dx=\left[e^xe^y\right]_0^2=e^2e^y-e^0e^y$$

$$\int_0^3\left[e^2e^y-e^y\right]dy=\left[e^2e^y-e^y\right]_0^3=\left[e^2e^3-e^3\right]-\left[e^2e^0-e^0\right]=e^5-e^3-e^2+1$$

Problem #25

$$\iint_D\left(x^2+y^2\right)dA$$

D is the triangular region whose vertices are (0, 0), (0, 1) and (1, 1).

$$\iint_D\left(x^2+y^2\right)dA=\int_0^1\left[\int_0^x\left(x^2+y^2\right)dy\right]dx$$

$$\int_0^x\left(x^2+y^2\right)dy=\left[x^2y+\frac{1}{3}y^3\right]_0^x=\left[x^2(x)+\frac{1}{3}x^3\right]-\left[x^2(0)+\frac{1}{3}0^3\right]=\frac{4}{3}x^3$$

$$\int_0^1\left[\int_0^x\left(x^2+y^2\right)dy\right]dx=\int_0^1\frac{4}{3}x^3\ dx=\left[\frac{4}{3}\left(\frac{1}{4}\right)x^4\right]_0^1=\frac{1}{3}\left(1^4\right)-0=\frac{1}{3}$$