March 28, 2007
Section 8.6

Problem #9

$$\iint_D xy\ dA; D = \{(x, y) : 0 \le x \le 2, 0 \le y \le 1\}$$

$$\int_0^1 \left[\int_0^2 xy\ dx \right] dy$$

$$\int_0^2 xy\ dx = \left[y \frac{1}{2} x^2 \right]_0^2 = \left[\frac{1}{2} x^2 y \right]_0^2 = \frac{1}{2}(2)^2 y - \frac{1}{2}(0)^2 y = 2y$$

$$\int_0^1 \left[\int_0^2 xy\ dx \right] dy = \int_0^1 [2y]\ dy = \left[y^2 \right]_0^1 = 1$$

Problem #15

$$\iint_D \frac{1}{xy}\ dA; D = \{(x, y) : 1 \le x \le 2, 1 \le y \le 3\}$$

$$\int_1^3 \left[\int_1^2 x^{-1} y^{-1}\ dx \right] dy$$

$$\int_1^2 x^{-1} y^{-1}\ dx = \left[y^{-1} \ln x \right]_1^2 = y^{-1} \ln 2 - y^{-1} \ln 1 = \frac{\ln 2}{y}$$

$$\int_1^3 \left[\int_1^2 x^{-1} y^{-1}\ dx \right] dy = \int_1^3 \left[\ln 2 \left(\frac{1}{y} \right) \right] dy = \left[(\ln 2)(\ln y) \right]_1^3$$

$$= \left[(\ln 2)(\ln 3) \right] - \left[(\ln 2)(\ln 1) \right] = (\ln 2)(\ln 3)$$

Problem #21

$$\iint_D xe^{x^2}\ dA; D = \{(x, y) : 0 \le x \le 2, 0 \le y \le 1\}$$

$$\int_0^1 \left[\int_0^2 xe^{x^2}\ dx \right] dy$$

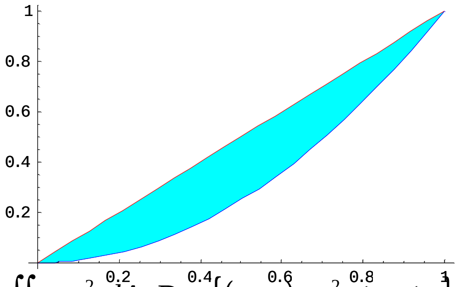
$$\int_0^2 xe^{x^2}\ dx = \frac{1}{2} \int_0^4 e^u\ du = \left[\frac{1}{2} e^u \right]_0^4 = \frac{1}{2} e^4 - \frac{1}{2} e^0 = \frac{1}{2} e^4 - \frac{1}{2}$$

		x	u
$u = x^2$			
$du = 2x\ dx$	UL	2	$2^2 = 4$
$\frac{1}{2} du = x\ dx$	LL	0	0

$$\int_0^1 \left[\int_0^2 xe^{x^2}\ dx \right] dy = \int_0^1 \left[\frac{1}{2} e^4 - \frac{1}{2} \right] dy = \left[\left(\frac{1}{2} e^4 - \frac{1}{2} \right) y \right]_0^1 = \left(\frac{1}{2} e^4 - \frac{1}{2} \right)(1) - \left(\frac{1}{2} e^4 - \frac{1}{2} \right)(0) = \left(\frac{1}{2} e^4 - \frac{1}{2} \right)$$

Problem #31

$f(x, y) = xy^2$, D is the region in the first quadrant bounded by $y = x$ and $y = x^2$



$$\iint_D xy^2\ dA;\ D=\left\{(x,y): x^2 \le y \le x\right\}$$

$$\int_0^1\left[\int_{x^2}^{x} xy^2\ dy\right]dx$$

$$\int_{x^2}^{x} xy^2\ dy=\left[x\left(\frac{1}{3}y^3\right)\right]_{x^2}^{x}=\frac{1}{3}x(x)^3-\frac{1}{3}x\left(x^2\right)^3=\frac{1}{3}\left(x^4-x^7\right)$$

$$\int_0^1\left[\int_{x^2}^{x} xy^2\ dy\right]dx=\frac{1}{3}\int_0^1\left(x^4-x^7\right)dx=\frac{1}{3}\left[\frac{1}{5}x^5-\frac{1}{8}x^8\right]_0^1=\frac{1}{3}\left[\frac{1}{5}-\frac{1}{8}\right]=\frac{1}{3}\left(\frac{3}{40}\right)=\frac{1}{40}$$