

March 1, 2007

Section 8.2

Problem #9

$$f(x, y) = e^{2x+3y}, \text{ point } (1, 1)$$

$$f_x = 2e^{2x+3y}; f_x(1, 1) = 2e^{2+3} = 2e^5$$

Slope of the curve in the x direction at the point (1, 1) is $2e^5$

$$f_y = 3e^{2x+3y}; f_y(1, 1) = 3e^5$$

Slope of the curve in the y direction at the point (1, 1) is $3e^5$

Problem #17

$$f(x, y) = \frac{1}{xy} = (xy)^{-1}, (1, 2)$$

$$\frac{\delta f}{\delta x} = -1(xy)^{-2} y = -\frac{y}{(xy)^2} = -\frac{y}{x^2 y^2} = -\frac{1}{x^2 y}$$

$$\frac{\delta f}{\delta x}(1, 2) = -\frac{1}{1^2(2)} = -\frac{1}{2}$$

$$\frac{\delta f}{\delta y} = -(xy)^{-2} x = -\frac{x}{x^2 y^2} = -\frac{1}{xy^2}$$

$$\frac{\delta f}{\delta y}(1, 2) = -\frac{1}{1(2^2)} = -\frac{1}{4}$$

Problem #19

$$f(x, y) = \frac{x-y}{x^2+y^2}, (2, 1)$$

$$f_x = \frac{(x^2+y^2)(1) - (x-y)(2x)}{(x^2+y^2)^2} = \frac{x^2+y^2-2x^2+2xy}{(x^2+y^2)^2} = \frac{y^2-x^2+2xy}{(x^2+y^2)^2}$$

$$f_x(2, 1) = \frac{1^2-2^2+2(2)(1)}{(2^2+1^2)^2} = \frac{1}{25}$$

$$f_y = \frac{(x^2+y^2)(-1) - (x-y)(2y)}{(x^2+y^2)^2} = \frac{-x^2-y^2-2xy+2y^2}{(x^2+y^2)^2} = \frac{y^2-x^2-2xy}{(x^2+y^2)^2}$$

$$f_y(2, 1) = \frac{1^2-2^2-2(2)(1)}{25} = \frac{-7}{25}$$

Example: $f(x, y) = 3x^2y^3 - 4x^4y$

Find $f_x = 6xy^3 - 16x^3y$

$$f_{xx} = 6y^3 - 48x^2y$$

$$f_{xy} = 18xy^2 - 16x^3$$

$$f_y = 9x^2y^2 - 4x^4$$

$$f_{yx} = 18xy^2 - 16x^3$$

$$f_{yy} = 18x^2y$$

Example:

$$f(x, y, z) = e^{2x+2y} - x^2z^3 + 4yz^5$$

$$\text{Want to find: } \frac{\delta^3 f}{\delta x \delta z \delta y} = \frac{\delta}{\delta x} \left[\frac{\delta}{\delta z} \right] \left[\frac{\delta f}{\delta y} \right] = \left[(f_y)_z \right]_x = f_{yzx}$$

$$\frac{\delta f}{\delta y} = 2e^{2x+2y} + 4z^5$$

$$\frac{\delta}{\delta z} \left[\frac{\delta f}{\delta y} \right] = \frac{\delta^2 f}{\delta z \delta y} = 20z^4$$

$$\frac{\delta}{\delta x} \left(\frac{\delta^2 f}{\delta z \delta y} \right) = 0$$