March 2, 2007
Section 8.2

Problem #7

$$f(x,y)=\sqrt{1+x^2y^2}; \quad \text{point } (1,0)$$

$$f(x,y)=\left(1+x^2y^2\right)^{1/2}$$

$$f_x=\frac{1}{2}\left(1+x^2y^2\right)^{-1/2}\left(2xy^2\right)$$

$$f_x(1,0)=\frac{1}{2}\left[1+1^2\left(0^2\right)\right]^{-1/2}\left[2(1)(0)^2\right]=0$$

$$f_y=\frac{1}{2}\left(1+x^2y^2\right)^{-1/2}\left(2x^2y\right)$$

$$f_y(1,0)=\frac{1}{2}\left[1+(1)^2(0)^2\right]^{-1/2}\left[2(1)^2 0\right]=0$$

Problem #9

$$f(x,y)=e^{2x+3y}; \quad \text{point } (1,1)$$

$$\frac{\delta f}{\delta x}=e^{2x+3y}(2)$$

$$\frac{\delta f}{\delta x}(1,1)=2e^{2+3}=2e^5$$

$$\frac{\delta f}{\delta y}=e^{2x+3y}(0+3)=3e^{2x+3y}$$

$$\frac{\delta f}{\delta y}(1,1)=3e^5$$

$$\frac{\delta^2 f}{\delta x\delta y}=\frac{\delta}{\delta x}\left(\frac{\delta f}{\delta y}\right)=\frac{\delta}{\delta x}\left(3e^{2x+3y}\right)=3e^{2x+3y}(2+0)=6e^{2x+3y}$$

$$\frac{\delta^2 f}{\delta x\delta y}=\left(f_y\right)_x=f_{yx}$$

Problem #11

$$f(x,y)=xye^{xy}$$

$$f_x=xye^{xy}(y)+e^{xy}(y)=xy^2e^{xy}+ye^{xy}$$

$$f_y=xye^{xy}(x)+e^{xy}(x)=x^2ye^{xy}+xe^{xy}$$

$$f_{xy}=\left(f_x\right)_y=\left(xy^2e^{xy}+ye^{xy}\right)_y$$

$$=\left[xy^2e^{xy}(x)+e^{xy}(2xy)\right]+\left[ye^{xy}(x)+e^{xy}(1)\right]$$

$$=x^2y^2e^{xy}+2xye^{xy}+xye^{xy}+e^{xy}$$

$$=x^2y^2e^{xy}+3xye^{xy}+e^{xy}$$

Problem #15

$$f(x,y)=e^{xy}\ln x$$

Find: $\dfrac{\delta^2 f}{\delta x\delta y}$

$$\frac{\delta f}{\delta y}=\ln x\left(e^{xy}\right)(x)=\left(x\ln x\right)e^{xy}$$

$$\frac{\delta^2 f}{\delta x\delta y}=\frac{\delta}{\delta x}\left[\left(x\ln x\right)e^{xy}\right]$$

$$=(x\ln x)\left(e^{xy}\right)_x+e^{xy}\left(x\ln x\right)_x$$

$$=\left(x\ln x\right)\left(e^{xy}y\right)+e^{xy}\left[x\left(\frac{1}{x}\right)+\ln x(1)\right]$$

$$=xy\ln xe^{xy}+e^{xy}\left[1+\ln x\right]$$

Example:

$$f\left(x,y,z\right)=3x^2y^3z^5$$

Find f_{yz}

$$f_y=9x^2y^2z^5$$

$$f_{yz}=\left(f_y\right)_z=45x^2y^2z^4$$

Find $\dfrac{\delta^2 f}{\delta x\delta y}$

$$\frac{\delta f}{\delta y}=9x^2y^2z^5$$

$$\frac{\delta^2 f}{\delta x\delta y}=\frac{\delta}{\delta x}\left(\frac{\delta f}{\delta y}\right)=18xy^2z^5$$