March 5, 2007

Section 8.2, Problem #21

$f(x, y) = x^2 y^4$

$f_x = 2xy^4$

$f_{xx} = 2y^4$

$f_{xy} = (2x)(4y^3) = 8xy^3$

$f_y = 4x^2y^3$

$f_{yx} = 8xy^3$

$f_{yy} = 12x^2y^2$

Section 8.1, Problem #35

$f(x, y) = 1 – x^2 – y^2$

Level curve $z_0 = 1$

$1 – x^2 – y^2 = 1$ or $x^2 + y^2 = 0$

point (0, 0)

Level curve at $z_0 = -1$

$1 – x^2 – y^2 = -1$ or $x^2 + y^2 = 2$

circle, center at origin, radius sqrt(2)

Practice Exam, #3a

$$\int_1^4 \left(e^{3x} - \frac{3}{x} \right) dx = \left[\frac{1}{3} e^{3x} - 3\ln|x| \right]_1^4 = \left[\frac{1}{3} e^{3(4)} - 3\ln|4| \right] - \left[\frac{1}{3} e^{3(1)} - 3\ln|1| \right]$$

$$= \frac{1}{3} e^{12} - 3\ln 4 - \frac{1}{3} e^{3}$$

Practice Test #5

$$f(x, y) = 2xye^{x+y} = (2xy)e^{x+y}$$

$$f_x = (2xy)e^{x+y}(1) + e^{x+y}(2y) = 2xye^{x+y} + 2ye^{x+y}$$

$$f_y = (2xy)e^{x+y}(1) + e^{x+y}(2x) = 2xye^{x+y} + 2xe^{x+y}$$

Integration by Parts

$$\int xe^{3x}\ dx$$

$u = x \qquad dv = e^{3x}\ dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

$$\int u\ dv = u\ v - \int v\ du$$

$$\int xe^{3x}\ dx = x\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x}\ dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\ dx$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{3}\left(\frac{1}{3}e^{3x}\right) + C = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

Section 7.4, Problem #4

$$\int_1^{\infty} \frac{1}{x^4}\,dx = \int_1^{\infty} x^{-4}\,dx = \lim_{b\to\infty}\int_1^{b} x^{-4}\,dx = \lim_{b\to\infty}\left[\frac{1}{-3}x^{-3}\right]_1^b = \lim_{b\to\infty}\left[\frac{-1}{3x^3}\right]_1^b$$

$$= \lim_{b\to\infty}\left[\frac{-1}{3b^3} - \frac{-1}{3(1)^3}\right] = \frac{1}{3}$$