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For tangent plane approximations (Sec. 8.5), I will give you the formula
In Chapter 9, sine and cosine only

Page 594, #21
minimize $z = 4x^2 + 2xy - 3y^2$ subject to constraint $x + y = 1$

$$F(x,y,\lambda) = 4x^2 + 2xy - 3y^2 + \lambda(x + y - 1)$$

$$F_x = 8x + 2y + \lambda = 0 \Rightarrow \lambda = -8x - 2y$$

$$F_y = 2x - 6y + \lambda = 0 \Rightarrow \lambda = 6y - 2x$$

$$F_\lambda = x + y - 1 = 0$$

$$-8x - 2y = 6y - 2x$$

$$-6x = 8y$$

$$x = -\frac{4}{3}y$$

$$-\frac{4}{3}y + y - 1 = 0$$

$$-\frac{1}{3}y = 1$$

$$y = -3$$

$$x = 4$$

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$$\int_0^1 \int_0^{\sqrt{x}} (x + y)\, dy\, dx$$

$$\int_0^{\sqrt{x}} (x + y)\, dy = \left[xy + \frac{1}{2}y^2\right]_0^{\sqrt{x}} = \left[x\sqrt{x} + \frac{1}{2}\left(\sqrt{x}\right)^2\right] - \left[x(0) + \frac{1}{2}(0)^2\right]$$

$$= x^{3/2} + \frac{1}{2}x$$

$$\int_0^1 \left(x^{3/2} + \frac{1}{2}x\right) dx = \left[\frac{2}{5}x^{5/2} + \frac{1}{4}x^2\right]_0^1 = \left[\frac{2}{5}(1)^{5/2} + \frac{1}{4}(1)^2\right] - \left[\frac{2}{5}(0)^{5/2} + \frac{1}{4}(0)^2\right]$$

$$= \frac{2}{5} + \frac{1}{4} = 0.4 + 0.25 = 0.65 = \frac{8+5}{20} = \frac{13}{20}$$

Page 594, #5
$z = f(x, y) = 16 - x^2 - y^2$
level curves corresponding to
$z_0 = 16$, we have $16 = 16 - x^2 - y^2$ or $x^2 + y^2 = 0$
$z_0 = 7$. we have $16 - x^2 - y^2 = 7$ or $9 = x^2 + y^2$

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$$f\left(x, y, z\right) = e^{-xyz} + xy^2z^3$$

$$f_x = e^{-xyz}\left(-yz\right) + y^2z^3 = -yze^{-xyz} + y^2z^3$$

$$f_y = e^{-xyz}\left(-xz\right) + xz^3\left(2y\right) = -xze^{-xyz} + 2xyz^3$$

$$f_z = -xye^{-xyz} + 3xy^2z^2$$

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$$\int_0^\infty x^2e^{-x^3}dx = \lim_{b\to\infty}\int_0^b x^2e^{-x^3}dx = \lim_{b\to\infty}\left[-\frac{1}{3}e^{-x^3}\right]_0^b = \lim_{b\to\infty}\left\{\left[-\frac{1}{3}e^{-b^3}\right]-\left[-\frac{1}{3}e^{-0^3}\right]\right\}$$

$$= \lim_{b\to\infty}\left[\frac{-1}{3e^{b^3}} + \frac{1}{3}\right] = 0 + \frac{1}{3} = \frac{1}{3}$$

$$\int x^2e^{-x^3}\ dx = -\frac{1}{3}\int e^u\ du = -\frac{1}{3}e^u + C = -\frac{1}{3}e^{-x^3} + C$$

$$u = -x^3$$

$$du = -3x^2\ dx$$

$$-\frac{1}{3}du = x^2\ dx$$

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$$y' = e^{2x},\ y(0) = 3$$

$$y = \int e^{2x}\ dx = \frac{1}{2}e^{2x} + C$$

$$x = 0,\ y = 3$$

$$3 = \frac{1}{2}e^0 + C$$

$$3 = \frac{1}{2} + C$$

$$C = \frac{5}{2}$$

$$y = \frac{1}{2}e^{2x} + \frac{5}{2}$$

Page 12.54, #9

$$\frac{dy}{dx} = xe^{x^2-y} = xe^{x^2}e^{-y}$$

$$\frac{1}{e^{-y}}\frac{dy}{dx} = xe^{x^2}$$

$$\int e^y\, dy = \int xe^{x^2}\, dx$$

$$e^y = \frac{1}{2}e^{x^2} + C$$

$$y = \ln\left[\frac{1}{2}e^{x^2} + C\right]$$

$$\int xe^{x^2}\, dx = \frac{1}{2}\int e^u\, du$$

$$u = x^2$$

$$du = 2x\, dx$$

$$\frac{1}{2}\, du = x\, dx$$