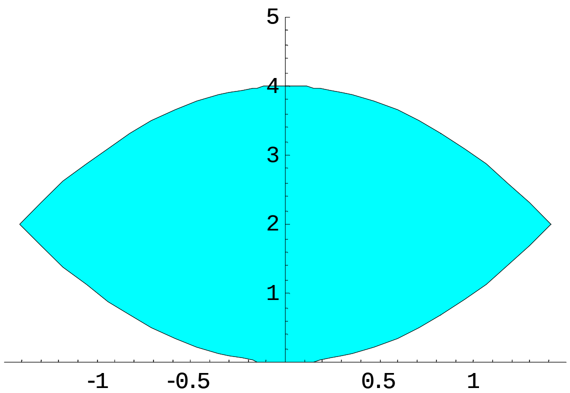
MATH 20
SAMPLE TEST #2
Questions, for the most part, are in bold.

1. Find the area enclosed by the curves y = x^2 and y = 4 – x^2.
Solution: The graph, which shows the area of interest is below.



The two curves intersect when

$$x^2 = 4 - x^2$$
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

The area becomes

$$\int_{-\sqrt{2}}^{\sqrt{2}} \left[\left(4 - x^2\right) - x^2\right] dx = \int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - 2x^2\right] dx = 2\cdot\int_0^{\sqrt{2}} \left[4 - 2x^2\right] dx = 2\left[4x - \frac{2}{3}x^3\right]_0^{\sqrt{2}}$$

$$= 2\left[2\sqrt{2} - \frac{2}{3}\left(\sqrt{2}\right)^3\right] = 2\left[2\sqrt{2} - \frac{2}{3}2\left(\sqrt{2}\right)\right] = 2\left[2\sqrt{2} - \frac{4}{3}\left(\sqrt{2}\right)\right] = \frac{4}{3}\left(\sqrt{2}\right)$$

2. Evaluate the following improper integral, if it is convergent. If it is not convergent, state that. Also, be sure to use the appropriate notation.
Solution:

$$\int_1^{\infty} \frac{x}{\left(x^2+1\right)^3}\, dx = \lim_{b\to\infty}\int_1^b \frac{x}{\left(x^2+1\right)^3}\, dx$$

$$\int \frac{x}{\left(x^2+1\right)^3}\, dx = \frac{1}{2}\int \frac{1}{u^3}du = \frac{1}{2}\int u^{-3}du = \frac{1}{2}\cdot\frac{-1}{2}u^{-2} = \frac{-1}{4u^2} = \frac{-1}{4\left(x^2+1\right)^2} + C$$

$$u = x^2 + 1$$
$$du = 2x\, dx$$
$$\frac{1}{2}du = x\, dx$$

$$\lim_{b\to\infty}\int_1^b \frac{x}{\left(x^2+1\right)^3}\, dx = \lim_{b\to\infty}\left[\frac{-1}{4\left(x^2+1\right)^2}\right]_0^b = \lim_{b\to\infty}\left[\frac{-1}{4\left(b^2+1\right)^2} - \frac{-1}{4\left(0^2+1\right)^2}\right] = \frac{1}{4}$$

3. Evaluate each of the following integrals:

$$a.\quad \int_1^4 \left(e^{3x} - \frac{3}{x}\right) dx = \left[\frac{1}{3}e^{3x} - \ln|x|\right]_1^4 = \left(\frac{1}{3}e^{12} - \ln|4|\right) - \left(\frac{1}{3}e^{3} - \ln|1|\right) = \frac{1}{3}e^{12} - \ln 4 - \frac{1}{3}e^{3}$$

$$b.\quad \int \frac{x^2}{\sqrt{x^3+1}}\, dx = \frac{1}{3}\int \frac{1}{\sqrt{u}} du = \frac{1}{3}\int u^{-1/2} du = \frac{1}{3}\frac{2}{1}u^{1/2} + C = \frac{2}{3}\sqrt{x^3+1} + C$$

$u = x^3 + 1$

$du = 3x^2\ dx$

$\frac{1}{3} du = x^2\ dx$

$$c.\quad \int x\sqrt{x+3}\ dx$$

$u = x \qquad dv = \sqrt{x+3}\ dx$

$du = dx \qquad v = \frac{2}{3}(x+3)^{3/2}$

$$\int x\sqrt{x+3}\ dx = x \cdot \frac{2}{3}(x+3)^{3/2} - \int \frac{2}{3}(x+3)^{3/2}\ dx = \frac{2}{3}x(x+3)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(x+3)^{5/2} + C$$

$$= \frac{2}{3}x(x+3)^{3/2} - \frac{4}{15}(x+3)^{5/2} + C$$

4. Consider the function $f(x,y) = \sqrt{4 - x^2 - y^2}$.

a. Evaluate the function at the point (1, -2).

$f(1,-2) = \sqrt{4 - 1^2 - (-2)^2} = \sqrt{4-1-4} = \sqrt{-1}$

Does not exist

b. What is the domain of this function?

$4 - x^2 - y^2 \geq 0$ or $4 \geq x^2 + y^2$

$D = \{(x,y) : x^2 + y^2 \leq 4\}$

c. Find the level curve for the function at z_0 = 1.

$\sqrt{4 - x^2 - y^2} = 1$

$4 - x^2 - y^2 = 1$

$3 = x^2 + y^2$

5. Consider the function f(x, y) = $2xye^{x+y}$. Find f_x and f_y.

$f_x = 2xy(e^{x+y}) + e^{x+y}(2y) = 2ye^{x+y}(x + 1)$

$f_y = 2xy(e^{x+y}) + e^{x+y}(2x) = 2xe^{x+y}(y + 1)$

6. Consider the function f(x, y, z) = $2xy^2 + 3xz^3 + 4yz^4$. Find $\frac{\partial^2 f}{\partial x \partial z}$

$$\frac{\partial f}{\partial z} = 9xz^2 + 8yz^3$$

$$\frac{\partial^2 f}{\partial x \partial z} = 9z^2$$