Section 12.1

Definition: A differential equation is any equation which has a derivative in it. A solution is a function which, when it and its appropriate derivatives are substituted into the equation, we get a true statement.

A boundary value (or initial value) problem is a differential equation and an initial condition.

Example: Show that $y = 3e^{2x}$ is a solution to the differential equation $y'' - 5y' + 6y = 0$
$y = 3e^{2x}$, $y' = 6e^{2x}$ and $y'' = 12e^{2x}$
Substitute that into $y'' - 5y' + 6y = 12e^{2x} - 5(6e^{2x}) + 6(3e^{2x}) = 12e^{2x} - 30e^{2x} + 18e^{2x} = 0$

Example: Solve the differential equation $\frac{dy}{dx} = x^2 + 2x$

The solution to this is just the antiderivative $\int \left(x^2 + 2x \right) dx = \frac{1}{3}x^3 + x^2 + C$

Example: If $y = Cx$ is a solution to the boundary value problem $y' = y/x$, $y(2) = 4$, find C.
$y(2) = 4$ means that when $x = 2$, $y = 4$
Check: Is $y = Cx$ a solution to $y' = y/x$?
$y = Cx$ means $y' = C$ and
$C = \frac{Cx}{x}$ which is true
To find C, let $x = 2$ and $y = 4$ in the solution $y = Cx$; so we have $4 = C(2)$ or $C = 2$
So, the solution to the boundary value problem $y' = y/x$, $y(2) = 4$ is $y = 2x$