Section 12.2

Definition: A differential equation is separable if it can be written as y'(x) = g(x)h(y)

$$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$$

$$\int \frac{1}{h(y)}\,dy = \int g(x)\,dx$$

Example: Solve y' = y/x

$$y' = \frac{dy}{dx} = \frac{y}{x} \Rightarrow y' = y\left(\frac{1}{x}\right)$$

$$\frac{1}{y}y' = \frac{1}{x}$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln y + a = \ln x + b$$

$$\ln y = \ln x + (b - a)$$

$$\ln y = \ln x + c$$

$$\ln y = \ln x + \ln C$$

$$\ln y = \ln Cx$$

$$y = Cx$$

Example: Solve

$$\frac{dy}{dx} = \frac{y^2}{x+1}$$

$$\frac{1}{y^2}\frac{dy}{dx} = \frac{1}{x+1}$$

$$\int y^{-2}\,dy = \int \frac{1}{x+1}\,dx$$

$$-y^{-1} = \ln(x+1) + c$$

$$\frac{-1}{y} = \ln(x+1) + \ln C$$

$$= \ln C(x+1)$$

$$\frac{y}{-1} = \frac{1}{\ln C(x+1)}$$

$$y = \frac{-1}{\ln\left[C(x+1)\right]}$$

Example: Solve $y' = \frac{xy^2 + x}{y}$, $y(0) = 3$

Solve:

$$y' = \frac{xy^2 + x}{y} = \frac{x\left(y^2+1\right)}{y} = x\left(\frac{y^2+1}{y}\right)$$

$$\frac{y}{y^2+1}\,y' = x$$

$$\int \frac{y}{y^2+1}\,dy = \int x\,dx$$

$$u = y^2 + 1$$

$$du = 2y\,dy$$

$$\frac{1}{2}\,du = y\,dy$$

$$\frac{1}{2}\int \frac{1}{u}\,du = \int x\,dx$$

$$\frac{1}{2}\ln u = \frac{1}{2}x^2 + c$$

$$\ln\left(y^2+1\right) = x^2 + 2c$$

$$e^{\ln\left(y^2+1\right)} = e^{x^2+2c} = e^{x^2}e^{2c}$$

$$y^2 + 1 = Ae^{x^2}$$

$$y^2 = Ae^{x^2} - 1$$

Boundary condition y(0) = 3

$$y^2 = Ae^{x^2} - 1$$

$$3^2 = Ae^0 - 1$$

$$A = 10$$

Answer: $y^2 = 10e^{x^2} - 1$