

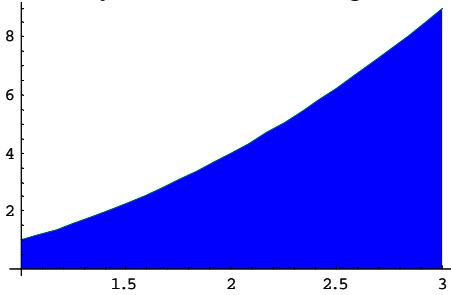
Section 6.6

Problem: Given two functions, $f(x)$ and $g(x)$, find the area bounded by the two curves with x in the interval from a to b .

Answer: $\int_a^b |f(x) - g(x)| dx$

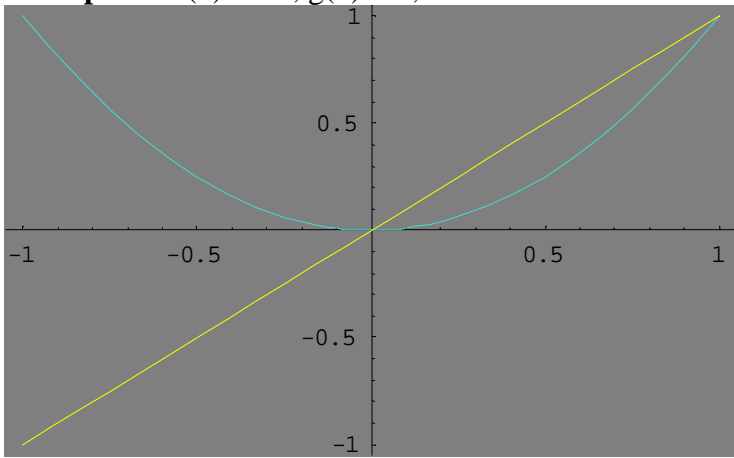
Example 1: $f(x) = x^2$, $y = 0$, $x = 1$ to $x = 3$

NOTE: $y = 0$ (x-axis), also $g(x) = 0$



$$\text{Area} = \int_1^3 x^2 dx = \frac{1}{3} x^3 \Big|_1^3 = \frac{1}{3} (3)^3 - \frac{1}{3} (1)^3 = 9 - \frac{1}{3} = \frac{26}{3}$$

Example 2: $f(x) = x^2$, $g(x) = x$, x from -1 to 1



$f(x) = x^2$ is turquoise and $g(x) = x$ is yellow. They intersect at $(0,)$

$$\begin{aligned} \int_{-1}^0 (x^2 - x) dx - \int_0^1 (x - x^2) dx &= \left[\frac{1}{3} x^3 - \frac{1}{2} x^2 \right]_{-1}^0 - \left[\frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_0^1 \\ &= [0] - \left[\frac{1}{3} (-1)^3 - \frac{1}{2} (-1)^2 \right] - \left\{ \left[\frac{1}{2} - \frac{1}{3} \right] - 0 \right\} = - \left[-\frac{1}{3} - \frac{1}{2} \right] - \left[\frac{1}{2} - \frac{1}{3} \right] = \frac{1}{3} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} = \frac{2}{3} \end{aligned}$$