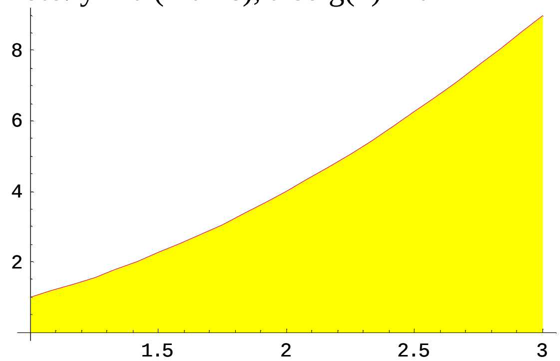
Section 6.6

Problem: Given two functions, f(x) and g(x), find the area bounded by the two curves with x in the interval from a to b.

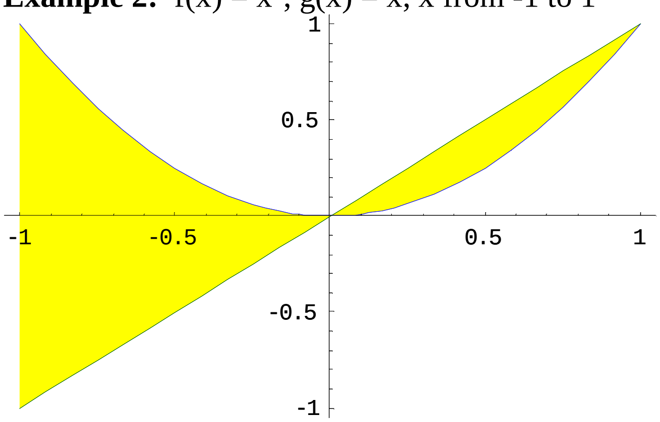
Answer: $\int_a^b |f(x) - g(x)| dx$

Example 1: Area of the region bounded by the curves $f(x) = x^2$, y = 0, x = 1 to x = 3
Note: y = 0 (x-axis), also g(x) = 0



$$\int_1^3 x^2\, dx = \left[\frac{1}{3}x^3\right]_1^3 = \frac{1}{3}(3)^3 - \frac{1}{3}(1)^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$$

Example 2: $f(x) = x^2$, g(x) = x, x from -1 to 1



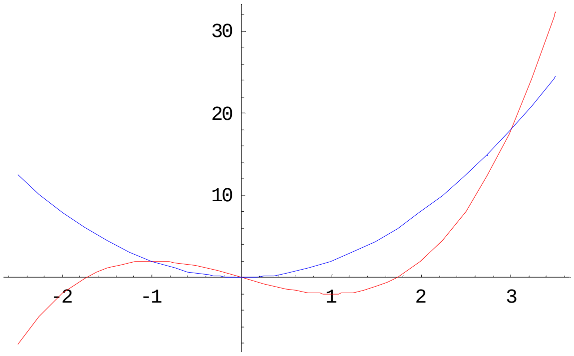
We need to do this in 2 parts, and the curves cross at the origin
The blue function is f(x); the green is g(x)

$$\int_{-1}^0 \left(x^2 - x\right) dx + \int_0^1 \left(x - x^2\right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_{-1}^0 + \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$$

$$= \left\{\left[\frac{1}{3}0^3 - \frac{1}{2}0^2\right] - \left[\frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2\right]\right\} + \left\{\left[\frac{1}{2}1^2 - \frac{1}{3}1^3\right] - \left[\frac{1}{2}0^2 - \frac{1}{3}0^3\right]\right\}$$

$$= -\left[-\frac{1}{3} - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{3} + \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1$$

Example 3: Find the area of the region bounded by the curves
$f(x) = x^3 - 3x$ and $g(x) = 2x^2$



So by observation, I see that they cross when x = -1 and when x = 3; also at x = 0
The blue curve is g(x) and the red curve is f(x)

$$\int_{-1}^{0}\left[\left(x^3-3x\right)-2x^2\right]dx+\int_{0}^{3}\left[2x^2-\left(x^3-3x\right)\right]dx$$

$$=\left[\frac{1}{4}x^4-\frac{3}{2}x^2-\frac{2}{3}x^3\right]_{-1}^{0}+\left[\frac{2}{3}x^3-\frac{1}{4}x^4+\frac{3}{2}x^2\right]_{0}^{3}$$

$$=\left\{[0]-\left[\frac{1}{4}(-1)^4-\frac{3}{2}(-1)^2-\frac{2}{3}(-1)^3\right]\right\}+\left\{\left[\frac{2}{3}3^3-\frac{1}{4}3^4+\frac{3}{2}3^2\right]-0\right\}$$

$$=-\left[\frac{1}{4}-\frac{3}{2}+\frac{2}{3}\right]+\left[\frac{54}{3}-\frac{81}{4}+\frac{27}{2}\right]=-\frac{82}{4}+\frac{30}{2}+\frac{52}{3}=-\frac{41}{2}+\frac{30}{2}+\frac{52}{3}=\frac{-123+90+104}{6}=\frac{71}{6}$$