Section 8.2

Partial Derivatives: Let z = f(x, y). Then the partial derivative of f with respect to x is the derivative when y is held constant. It is written $f_x = \frac{\delta f}{\delta x}$. The partial of f with respect to y is the derivative when x is held constant.

It is written $\frac{\delta f}{\delta y} = f_y$.

Note: $\frac{\partial f}{\partial x}(a,b)$ is the slope of the line tangent to the curve z = f(x, y) at the point (a, b) moving parallel to the x-axis. $\frac{\partial f}{\partial y}(a,b)$ is the slope of the line tangent to the curve z = f(x, y) at the point

(a, b) moving parallel to the y-axis. We do not consider slopes of tangent lines moving in any other direction.

Example: Let P(s, l) be the profit in hundreds of dollars if I sell s snow blowers and l lawn mowers. Then $\frac{\delta P}{\delta s}$ = P_s would tell me how changing the number of snow blowers sold when the numbers of lawnmowers does not change that changes the profit. Likewise, $\frac{\delta P}{\delta l}$ = P_l tells me how profits change if the number of lawn mowers changes but the number of snow blowers is constant.

Example: f(x, y) = $2x^2 + 3xy - 4y^2$

Find $f_x(-1, 1)$ and $f_y(2, -3)$

$f_x = 4x + 3y$; $f_x(-1, 1) = -4 + 3 = -1$

$f_y = 3x - 8y$; $f_y(2, -3) = 6 + 24 = 30$

Second Order Partial Derivatives:

$f(x, y)$

First-order partial derivatives

$\frac{\delta f}{\delta x} = f_x$ and $\frac{\delta f}{\delta y} = f_y$

We can take the derivative of each of these:

$$\frac{\delta}{\delta x}\left[\frac{\delta f}{\delta x}\right] = \frac{\delta^2 f}{\delta x^2} \qquad \frac{\delta}{\delta y}\left[\frac{\delta f}{\delta x}\right] = \frac{\delta^2 f}{\delta y \delta x}$$

$$[f_x]_x = f_{xx} \qquad [f_x]_y = f_{xy}$$

also,

$$\frac{\delta}{\delta y}\left[\frac{\delta f}{\delta y}\right] = \frac{\delta^2 f}{\delta y^2} \qquad \frac{\delta}{\delta x}\left[\frac{\delta f}{\delta y}\right] = \frac{\delta^2 f}{\delta x \delta y}$$

$$\left[f_y\right]_y = f_{yy} \qquad \left[f_y\right]_x = f_{yx}$$

Example: $f(x, y) = x^3 + 3x^2y – 4y^4$

Find f_{xx} and f_{xy}

First derivative with respect to x: $f_x = 3x^2 + 6xy$

Second derivatives:

With respect to x: $(f_x)_x = f_{xx} = 6x + 6y$

With respect to y: $(f_x)_y = 6x$

Find f_{yy} and f_{yx}

Step 1: Find f_y

$f_y = 0 + 3x^2 – 16y^3$

Step 2: Take the derivative of this result

With respect to y: $f_{yy} = -48y^2$

With respect to x: $f_{yx} = 6x$