

Section 11.1

Experiment Anything which we can replicate and predict all of the possible outcomes
 Outcomes
 Sample Space—List of outcomes
 Event—Subset of the sample space

Example: Experiment: Suppose that we roll a single die.
 Sample Space {1, 2, 3, 4, 5, 6}
 Possible events: {1}, or {2, 4, 6}, {1, 2, 3}, ...

The Probability of an Event E

If each outcome is equally likely, then $p(E) = n(E) / n(S)$

Continue Example: $n(S) = 6$

$p(\{1, 2\}) = 1/6$

$p(\text{even}) = 3/6$

Properties of Probability: Let E be an event in a sample space S. Also, let A and B be two events in S. Then

1. $0 \leq p(E) \leq 1$
2. $p(S) = 1$
3. $p(\emptyset) = 0$
4. $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
5. $p(E^c) = 1 - p(E)$
6. $p(A \cup B) = p(A) + p(B), \quad A \cap B = \emptyset$

Example: A bag contains 3 balls. They are identical except that they are numbered 1, 2, and 3. A person picks a ball at random, notes the number, and returns the ball to the bag. They then pick a ball and record the sum of the numbers on the two balls.

1. What is the sample space?
2. What is the probability of each outcome?
3. What is the probability distribution of the experiment?
4. What is the mean of this experiment? NOTE: This is also called the expected value of this experiment.
5. What is the variance and standard deviation of this experiment?

1. Sample Space—Fill the sums of the balls into the table

		Ball 2		
		1	2	3
Ball 1	1	2	3	4
	2	3	4	5
	3	4	5	6

So I have a total of 9 outcomes, all of which are equally likely.

3. Probability distribution gives each of the outcomes and the associated probability.

Outcomes	2	3	4	5	6
Probability	1/9	2/9	3/9	2/9	1/9

4. Mean (arithmetic average) of this experiment

$$2(1/9) + 3(2/9) + 4(3/9) + 5(2/9) + 6(1/9) = 4$$

If we did this experiment many times, and average the outcomes, we would expect an average of about

4.

The mean is also called the expected value.

5. Variance and standard deviation

$$\begin{aligned} & \sum (\text{outcome-expectedvalue})^2 \text{Probability} \\ & = (2-4)^2 \frac{1}{9} + (3-4)^2 \frac{2}{9} + (4-4)^2 \frac{3}{9} + (5-4)^2 \frac{2}{9} + (6-4)^2 \frac{1}{9} = \frac{4}{3} \end{aligned}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$