Section 11.1

Experiment Anything which we can replicate and predict all of the possible outcomes

Roll a die

Outcomes; 1, 2, 3, 4, 5, 6
Sample Space—List of outcomes {1, 2, 3, 4, 5, 6}
Event—Subset of the sample space
Possible events: {1}, or {2, 4, 6}, {1, 2, 3}, …

The Probability of an Event E

If each outcome is equally likely, then p(E) = n(E) / n(S)

Continue to talk about the die: n(S) = 6

p({1, 2, 3}) = 3/6
p(even) = 3/6
p({1}) = 1/6
Let A = {1, 2, 3} and B = {2, 4, 6}. A U B = {1, 2, 3, 4, 6}

$$p(A \cup B) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$$

$$A \cap B = \{2\}$$

E^C is called the compliment of E; it is all of the elements of the sample space that are not in E. So, if E = {1, 2}, E^C = {3, 4, 5, 6}.

$p(E^C) = 4/6 = 1 - 2/6 = 1 - p(E)$

Properties of Probability: Let E be an event in a sample space S. Also, let A and B be two events in S. Then

1. $0 \le p(E) \le 1$
2. $p(S) = 1$ (certainty)
3. $p(\varnothing) = 0$ (impossible)
4. $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
5. $p(E^c) = 1 - p(E)$
6. $p(A \cup B) = p(A) + p(B), \quad A \cap B = \varnothing$

Example: A bag contains 3 balls. They are identical except that they are numbered 1, 2, and 4. A person picks a ball at random, notes the number, and returns the ball to the bag. They then pick a ball and record the sum of the numbers on the two balls.

1. What is the sample space?
2. What is the probability of each outcome?
3. What is the probability distribution of the experiment?
4. What is the mean of this experiment? NOTE: This is also called the expected value of this experiment.
5. What is the variance and standard deviation of this experiment?

1. Sample Space—Fill the sums of the balls into the table

		Ball 2		
		1	2	4
Ball 1	1	**2**	**3**	**5**

	2	**3**	**4**	**6**
	4	**5**	**6**	**8**

So I have a total of 9 outcomes, all of which are equally likely.

3. What is the probability distribution of the experiment?

Outcomes	2	3	4	5	6	8
Probability	1/9	2/9	1/9	2/9	2/9	1/9

4. Mean (arithmetic average)of this experiment

$$\frac{2+3+5+3+4+6+5+6+8}{9}=\frac{2}{9}+\frac{3+3}{9}+\frac{4}{9}+\frac{5+5}{9}+\frac{6+6}{9}+\frac{8}{9}$$

$$=2\left(\frac{1}{9}\right)+3\left(\frac{2}{9}\right)+4\left(\frac{1}{9}\right)+5\left(\frac{2}{9}\right)+6\left(\frac{2}{9}\right)+8\left(\frac{1}{9}\right)=\frac{42}{9}=\frac{14}{3}$$

$$mean=\sum outcome\times probability$$

If we did this experiment many times and find the average of all of these outcomes, I would expect the average to be about 14/3

We call the mean the expected value.

5. Variance and standard deviation

$$\sum(\text{outcome-expectedvalue})^2\text{Probability}$$

Variance

$$Var=\left(2-\frac{14}{3}\right)^2\frac{1}{9}+\left(3-\frac{14}{3}\right)^2\frac{2}{9}+\left(4-\frac{14}{3}\right)^2\frac{1}{9}+\left(5-\frac{14}{3}\right)^2\frac{2}{9}+\left(6-\frac{14}{3}\right)^2\frac{2}{9}+\left(8-\frac{14}{3}\right)^2\frac{1}{9}$$

$$=\left(-\frac{8}{3}\right)^2\frac{1}{9}+\left(-\frac{5}{3}\right)^2\frac{2}{9}+\left(-\frac{2}{3}\right)^2\frac{1}{9}+\left(\frac{1}{3}\right)^2\frac{2}{9}+\left(\frac{4}{3}\right)^2\frac{2}{9}+\left(\frac{10}{3}\right)^2\frac{1}{9}$$

$$=\frac{64}{81}+\frac{50}{81}+\frac{4}{81}+\frac{2}{81}+\frac{32}{81}+\frac{100}{81}=\frac{252}{81}=\frac{28}{9}$$

Standard Deviation = $\sqrt{Var}=\sqrt{\frac{28}{9}}=\frac{\sqrt{28}}{3}=\frac{2\sqrt{7}}{3}$

We call this a measure of the dispersion.