Section 8.3

Problem: Find all points where the function $f(x, y) = -x^2 + y^3 + 6x - 12y$ has a relative extreme point and determine if it is a relative maximum or relative minimum.

Step 1: Find critical points

a. Find f_x and f_y

$f_x = -2x + 6$

$f_y = 3y^2 - 12$

b. Solve $f_x = 0$, $f_y = 0$

$-2x + 6 = 0$ So, $x = 3$

$3y^2 - 12 = 0$ So $y^2 = 4$ and $y = -2, +2$

Critical points are (3, -2) and (3, + 2)

Step 2: Use the Second Derivative Test

Define $D(x, y) = f_{xx}f_{yy} - (f_{xy})^2$. Let (a, b) be a critical point.

1. If $D(a, b) < 0$, the point is called a saddle point
2. If $D(a, b) > 0$,
   a. If $f_{xx}(a, b) > 0$, then (a, b) is a relative minimum.
   b. If $f_{xx}(a, b) < 0$, then (a, b) is a relative maximum.

a. Find f_{xx}, f_{yy}, f_{xy}

$f_x = -2x + 6$ $\qquad f_{xx} = -2$

$f_y = 3y^2 - 12$ $\qquad f_{yy} = 6y$

$f_{xy} = 0$

b. Find $D(x, y) = (-2)(6y) - 0^2 = -12y$

c. Use that for each critical point

i. For (3, -2), $D(3, -2) = -12(-2) = 24$ and $f_{xx} = -2$

Relative maximum at (3, -2)

ii. For the point (3, +2), $D(3, 2) = -12(2) = -24$

Saddle point (3, 2)