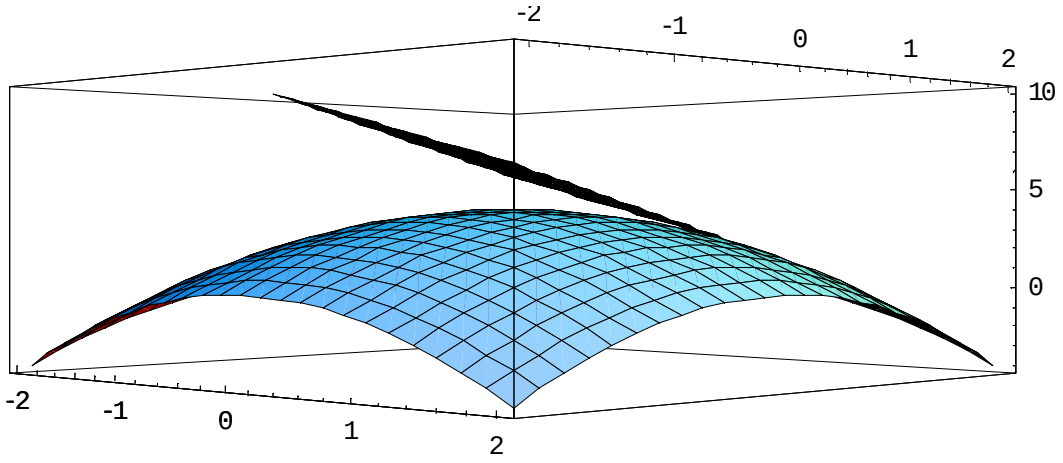
Lecture: Section 8.5

Reminder: If z = f(x, y), the partial derivatives help us determine the change in z if one variable changes and the other variable is a constant. What if we want a change in both variables?

This is the graph of $z = f(x, y) = 4 - x^2 - y^2$ showing the plane that is tangent to the curve at the point (1, 1). For a small change in x and y, we could use this plane to approximate the change in z.



Tangent Plane Approximations: Suppose that z = f(x, y) and $f_x(a, b)$ and $f_y(a, b)$ both exist. If the changes in x and y $(\Delta x \text{ and } \Delta y, \text{ respectively})$ are both small, then

$$\Delta z = f(a+\Delta x, b+\Delta y) - f(a,b) \approx f_x(a,b)\Delta x + f_y(a,b)\Delta y = dz$$

Example: $f(x, y) = 4 - x^2 - y^2$ with a = 1, b = 1

$f_x = -2x$ and $f_y = -2y$

Δx	Δy	$f(a+\Delta x, b+\Delta y)$	$\Delta z = f(a+\Delta x, b+\Delta b) - f(a,b)$	$dz = f_x(1,1)\Delta x + f_y(1,1)\Delta y$
0.1	0.05	f(1.1, 1.05) = 1.6875	1.6875 – f(1, 1) = -0.3125	(-2)(0.1) + (-2)(0.05) = -0.3
0.2	0.3	f(1.2, 1.3) = 0.87	0.87 – f(1, 1) = -1.13	(-2)(0.2) + (-2)(0.3) = -1.0

In moving from the point (1, 1) to (1.2, 1.3), z decreases by an amount of 1.13; our approximation says the change is a decrease of 1.

Both changes are fairly small so the approximations are not too bad.

Another Example:

Let $z = f(x, y) = x^2 - 3xy + 2y^2$

Let's find the change in z if x changes from 1 to 1.2 and y changes from 2 to 1.9

a = 1, b = 2, $\Delta x = 0.2$, $\Delta y = -0.1$

$f_x = 2x - 3y$; $f_x(1, 2) = 2 - 6 = -4$

$f_y = -3x + 4y$; $f_y(1 , 2) = -3 + 8 = 5$

$$dz = f_x(1,2)\Delta x + f_y(1,2)\Delta y$$

$$= (-4)(0.2) + (5)(-0.1) = -1.3$$

How good is this approximation?

$\Delta z = f\left(a+\Delta x, b+\Delta b\right) - f\left(a,b\right) = f(1.2,1.9) - f(1,2) = -1.18$

What is the error? actual – approximation = -1.18 – (-1.3) = 0.12

relative error = $\left|\frac{error}{actual}\right| = \left|\frac{0.12}{-1.18}\right| = 0.1016$

per cent error= relative error x 100% = 10.1%