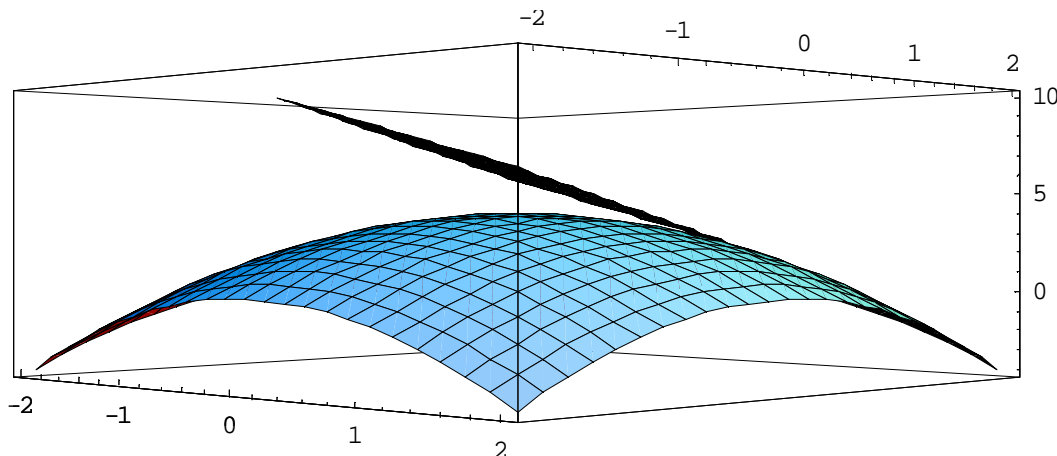


**Lecture:** Section 8.5

Reminder: If  $z = f(x, y)$ , the partial derivatives help us determine the change in  $z$  if one variable changes and the other variable is a constant. What if we want a change in both variables?

This is the graph of  $z = f(x, y) = 4 - x^2 - y^2$  showing the plane that is tangent to the curve at the point  $(1, 1)$ . For a small change in  $x$  and  $y$ , we could use this plane to approximate the change in  $z$ .



**Tangent Plane Approximations:** Suppose that  $z = f(x, y)$  and  $f_x(a, b)$  and  $f_y(a, b)$  both exist. If the changes in  $x$  and  $y$  ( $\Delta x$  and  $\Delta y$ , respectively) are both small, then

$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b) \approx f_x(a, b)\Delta x + f_y(a, b)\Delta y = dz$$

**Example:**  $f(x, y) = 4 - x^2 - y^2$  with  $a = 1, b = 1$   
 $f(1, 1) = 2$   
 $f_x = -2x, f_x(1, 1) = -2$   
 $f_y = -2y, f_y(1, 1) = -2$

$\Delta x$	$\Delta y$	$f(a + \Delta x, b + \Delta y)$	$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$ actual change	$dz = f_x(1,1)\Delta x + f_y(1,1)\Delta y$ approximation
0.1	0.05	$f(1.1, 1.05) = 1.6875$	$1.6875 - 2 = -0.3175$	$(-2)(0.1) + (-2)(0.05) = -0.3$
0.2	0.3	$f(1.2, 1.3) = 0.87$	$0.87 - 2 = -1.13$	$(-2)(0.2) + (-2)(0.3) = -1$

In moving from  $(1, 1)$  to the point  $(1.1, 1.05)$  we had an actual change of  $-0.3175$  and an approximate change of  $-0.3$ . This was an error of  $0.0175$ . Relative error =  $\left| \frac{\text{error}}{\text{actual}} \right| = \left| \frac{0.0175}{-0.3175} \right| = 0.055$  and the percent error is  $5.5\%$

In moving from the point  $(1, 1)$  to  $(1.2, 1.3)$ ,  $z$  decreases by an amount of  $1.13$ ; our approximation says the change is a decrease of  $1$ .

Relative error =  $0.13/1.13 = 0.115$  or a percent error of  $11.5\%$

Both changes are fairly small so the approximations are not too bad.

Same function, but start at  $x = 4, y = 8$  and have  $\Delta x = 0.2$  and  $\Delta y = 0.3$   
 $f(4, 8) = -76$ ,  
 actual error =  $\Delta z = f(4.2, 8.3) - f(4, 8) = -6.53$

$$\text{approximation} = f_x(4, 8)(0.2) + f_y(4, 8)(0.3) = -2(4)(0.2) + (-2)(8)(0.3) = -6.4$$