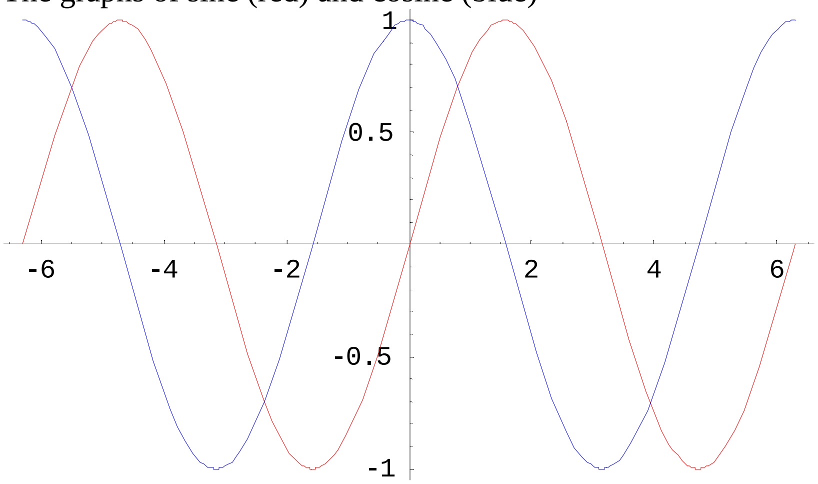
Section 9.3

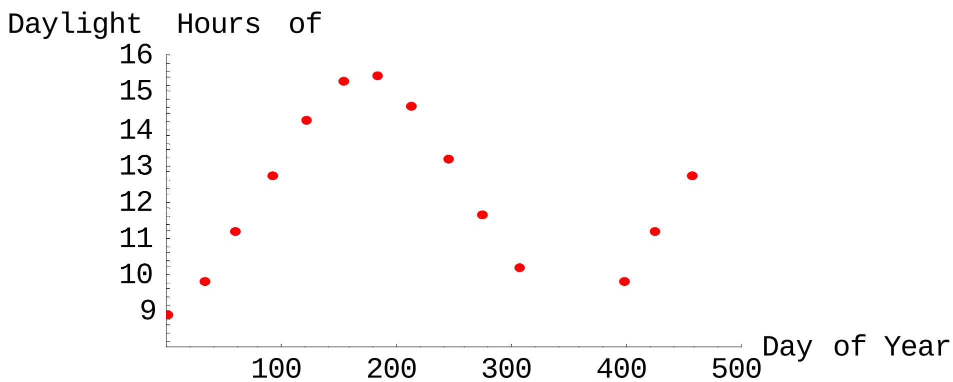
The graphs of sine (red) and cosine (blue)



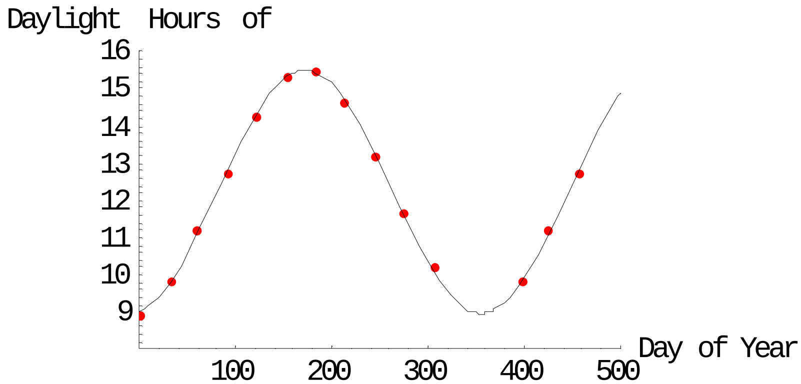
The number of hours of daylight in Burlington, Vermont is given in the table below.

Date	Jan 1	Feb 1	Mar 1	Apr 1	May 1	Jun 1	Jul 1	Aug 1	Sep 1
Day of Year	1	32	60	90	121	152	182	213	244
Number of Hours	8.92	9.85	11.18	12.77	14.23	15.33	15.48	14.63	13.22

Date	Oct 1	Nov 1	Dec 1
Day of Year	274	305	335
Number of Hours	11.70	10.20	9.08



$f(t) = 3.263 \sin(0.0172t - 1.385) + 12.230$ hours of daylight on the t^{th} day of the year



Derivatives of the sine and cosine functions:

$$\frac{d}{dt}\sin[f(t)] = \cos[f(t)]\,f'(t)$$

$$\frac{d}{dt}\cos[f(t)] = -\sin[f(t)]\,f'(t)$$

Examples: Differentiate each of the following:

1. $y = \cos(x^3)$

   $y' = [-\sin(x^3)](3x^2) = -3x^2 \sin(x^3)$

2. $f(x) = x^2 \sin(4x - 7)$

   $f'(x) = x^2 [4\cos(4x - 7)] + \sin(4x - 7)[2x]$

3. $g(t) = \dfrac{\sin 2t}{\cos 3t}$

$$\frac{dg}{dt} = \frac{\cos 3t(\cos 2t)(2) - \sin 2t(-\sin 3t)(3)}{\cos^2 3t} = \frac{2\cos 2t \cos 3t + 3\sin 2t \sin 3t}{\cos^2 3t}$$

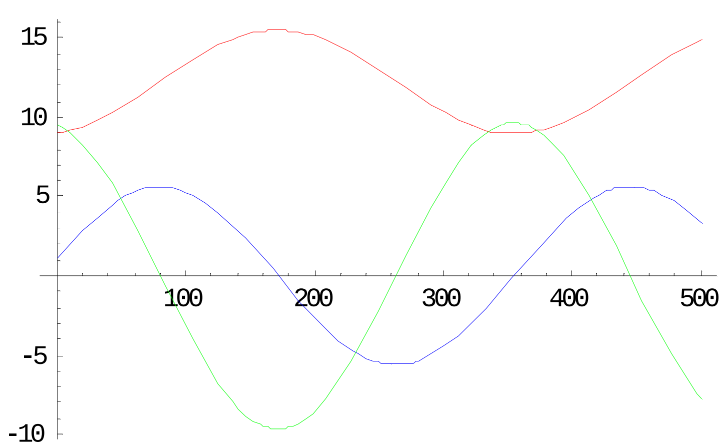
Example:

$f(t) = 3.263 \sin(0.0172t - 1.385) + 12.230$ hours of daylight on the t^{th} day of the year

$f'(t) = 3.263 \cos(0.0172t - 1.385)(0.0172)$

$f''(t) = -3.263(0.0172)\sin(0.0172t - 1.385)(0.0172)$

NOTE: On the graph below, f(t) is red, f '(t) is blue, and f ''(t) is green.



f ‘(t) = 0 on days 171.85 and 354.5

Day 172 is June 21 and day 354 is December 22

The day with the greatest number of hours of daylight is June 21 and the day with the least number of hours of daylight is December 22

f ‘’(t) = 0 on days 80.2 and 263.17

Day 80 is March 20 and Day 263 is September 20. These are the first day of Spring and Fall.

On March 20, the number of hours of daylight is increasing the fastest. On September 20, the number of hours of daylight is decreasing at the fastest rate.

Some more examples:

Find the derivative of each of the following functions:

$y = \sin(\ln x)$

$$\frac{dy}{dx} = \cos(\ln x)\left(\frac{1}{x}\right) = \frac{\cos(\ln x)}{x}$$

$f(t) = e^{\cos t}$

$f'(t) = e^{\cos t}\left(-\sin t\right)$

Find the equation of the line tangent to the curve f(x) = cos (2x + π/3) at x = π/6

Need a point: x = π/6; y = $\cos\left(2\left(\frac{\pi}{6}\right)+\frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

Need slope:

$$f'(x) = -\sin\left(2x+\frac{\pi}{3}\right)(2) = -2\sin\left(2x+\frac{\pi}{3}\right)$$

$$m = f'\left(\frac{\pi}{6}\right) = -2\sin\left(\frac{2\pi}{3}\right) = -2\left(\frac{\sqrt{3}}{2}\right) - \sqrt{3}$$

Equation:

$$y - \left(-\frac{1}{2}\right) = -\sqrt{3}\left(x - \frac{\pi}{6}\right)$$