Section 9.4

Integrals of the sine and cosine functions

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

Examples: Integrate the following:

$$\int \sin 5x\, dx$$

$$u = 5x$$

$$du = 5\, dx$$

$$\frac{1}{5}\, du = dx$$

$$\int \sin 5x\, dx = \frac{1}{5}\int \sin u\, du = \frac{1}{5}(-\cos u) + C = -\frac{1}{5}\cos 5x + C$$

$$\int \sqrt{\cos 2x}\ \sin 2x\, dx$$

$$u = \cos 2x$$

$$du = -2\sin 2x\, dx$$

$$-\frac{1}{2}\, du = \sin 2x\, dx$$

$$\int \sqrt{\cos 2x}\ \sin 2x\, dx = \int \sqrt{u}\ \left(\frac{-1}{2}\right) du = -\frac{1}{2}\int u^{1/2}\, du = -\frac{1}{2}\left(\frac{2}{3}\right)u^{3/2} + C$$

$$= -\frac{1}{3}(\cos 2x)^{3/2} + C$$

$$\int x \sin x\, dx$$

$$u = x \qquad dv = \sin x\, dx$$

$$du = dx \qquad v = -\cos x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \sin x\, dx = x(-\cos x) - \int (-\cos x)\, dx = -x\cos x + \int \cos x\, dx$$

$$= -x\cos x + \sin x + C$$