MATH 20
TEST #1 SAMPLE

NOTE: This is a sample to give you an idea of the types of questions that might be on your TEST 1.

There are a total of 6 problems on 4 pages. Please be sure that you have the entire test. Also, show all necessary work; answers that seem to appear by magic will receive no credit.

1. Differentiate the following functions:

$$a.\ \ f(x)=\frac{x^2}{x+2}$$

$$f'(x)=\frac{(x+2)(2x)-x^2}{(x+2)^2}=\frac{2x^2+4x-x^2}{(x+2)^2}=\frac{x^2+4x}{(x+2)^2}$$

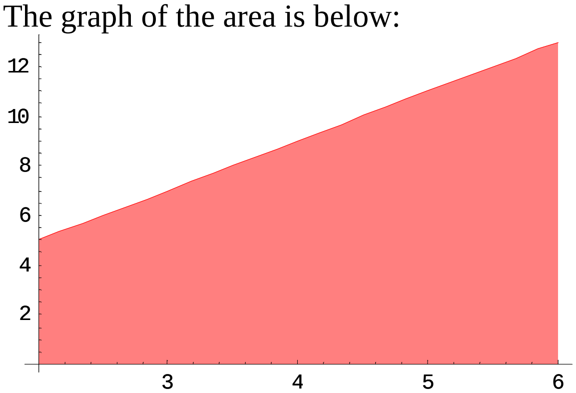
$$b.\ y=\ln\left(3x^2+e^{5x}\right)$$

$$\frac{dy}{dx}=\frac{6x+5e^{5x}}{3x^2+e^{5x}}$$

2. Use geometry to evaluate the definite integral

$$\int_2^6 (2t+1)dt.$$

The graph of the area is below:



This is a rectangle (base = 4, height = 5) topped by a triangle (base = 4, height = 8); area is 20 + 16 = 36

3. Find each of the following antiderivatives:

$$a.\ \int\left(\frac{5}{y^2}-4y^2\right)dy=\int\left(5y^{-2}-4y^2\right)dy=5\left(\frac{1}{-1}\right)y^{-1}-4\left(\frac{1}{3}\right)y^3+C=\frac{-5}{y}-\frac{4}{3}y^3+C$$

$$b.\int(x+3)\left(x^2+6x+2\right)^4 dx$$

$$u=x^2+6x+2$$

$$du=(2x+6)\ dx$$

$$\frac{1}{2}\ du=(x+3)\ dx$$

$$\int(x+3)\left(x^2+6x+2\right)^4 dx=\frac{1}{2}\int u^4\ du=\frac{1}{2}\left(\frac{1}{5}\right)u^5+C=\frac{1}{10}\left(x^2+6x+2\right)+C$$

$$c.\ \int \frac{\sqrt{x^{1/2}-3}}{\sqrt{x}}\,dx$$

$$u = x^{1/2} - 3$$

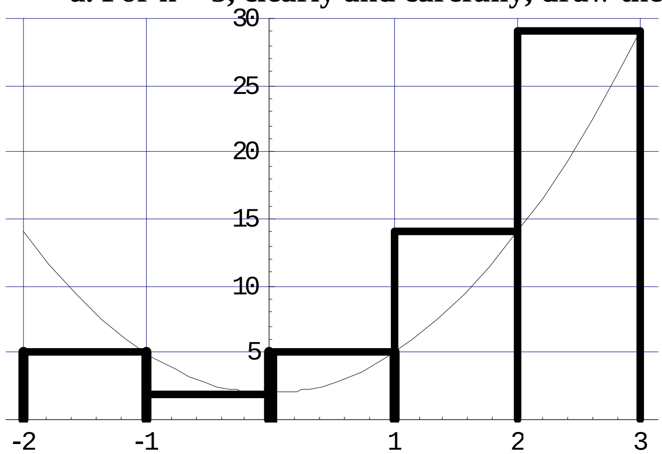
$$du = \frac{1}{2\sqrt{x}}\,dx$$

$$2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$\int \frac{\sqrt{x^{1/2}-3}}{\sqrt{x}}\,dx = 2\int \sqrt{u}\,du = 2\left(\frac{2}{3}\right)u^{3/2} + C = \frac{4}{3}\left(x^{1/2}-3\right)^{3/2} + C$$

4. The following is a graph of f(x) = 3x^2 + 2 on the interval [-2, 3].

a. For n = 5, clearly and carefully, draw the right-hand rectangles on the graph.



b. Find the right-hand sum.

rectangle	1	2	3	4	5
right-hand endpoint	-1	0	1	2	3
height (function)	f(-1) = 5	f(0) = 2	5	14	29
width	1	1	1	1	1
area	5	2	5	14	29

Sum is 5 + 2 + 5 + 14 + 29 = 55

c. If v = f(t) is a velocity in feet per second and t is time in seconds, what does the answer to part b tell us?

In the time from t = 0 to t = 5, the object travels about 55 feet.

5. Use the Fundamental Theorem of Calculus to evaluate

$$\int_{-1}^{1}(2x+1)^3\,dx$$

$u = 2x+1$		x	$u = 2x+1$
$du = 2\,dx$	upper limit	1	$2(1)+1=3$
$\frac{1}{2}\,du = dx$	lower limit	-1	$2(-1)+1=-1$

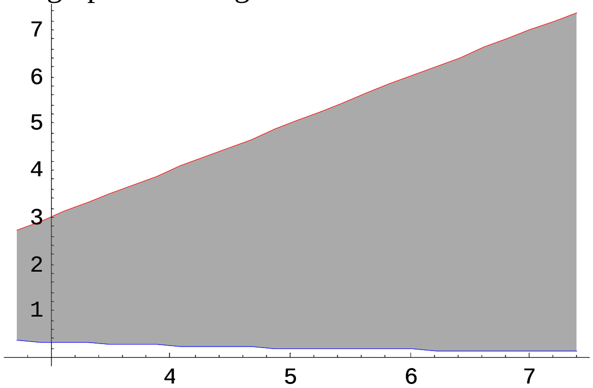
$$\int_{-1}^{1}(2x+1)^3\,dx = \frac{1}{2}\int_{-1}^{3}u^3\,du = \left[\frac{1}{2}\left(\frac{1}{4}\right)u^4\right]_{-1}^{3} = \frac{1}{8}(3)^4 - \frac{1}{8}(-1)^4 = \frac{81}{8} - \frac{1}{8} = \frac{80}{8} = 10$$

6. Find the average value of the function f(x) = x^2 on the interval [0, 4].

$$\frac{1}{4-0}\int_{0}^{4}x^2\,dx = \left[\frac{1}{4}\left(\frac{1}{3}\right)x^3\right]_{0}^{4} = \frac{1}{12}4^3 - \frac{1}{12}0^3 = \frac{16}{3}$$

7. Find the area enclosed by the curves y = x, y = 1/x, x = e and x = e^2.

The graph of the region is below:



The red curve is y = x and the blue curve is y = 1/x. The left endpoint is e and the right is e^2.

$$\int_{e}^{e^2}\left(x - \frac{1}{x}\right)dx = \left[\frac{1}{2}x^2 - \ln x\right]_{e}^{e^2} = \left[\frac{1}{2}\left(e^2\right)^2 - \ln e^2\right] - \left[\frac{1}{2}e^2 - \ln e\right] = \frac{1}{2}e^4 - 2 - \frac{1}{2}e + 1 = \frac{1}{2}e^4 - 1 - \frac{1}{2}e$$