NOTE: There are two forms of this test; be sure that you have the correct form when you check you solutions.

FIRST FORM

1. Evaluate the following integral: $\int x^2 \ln x dx$ **.**

$$\int x^2 \ln x dx$$

$$u = \ln x \qquad dv = x^2\ dx$$

$$du = \frac{1}{x}\, dx \qquad v = \frac{1}{3}x^3$$

$$\int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \frac{1}{x}\, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\ dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right) + C$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

2. Evaluate the following integral: $\int 3x\, e^{2x}\ dx$ **.**

$$\int 3x\, e^{2x}\ dx$$

$$u = 3x \qquad dv = e^{2x}\ dx$$

$$du = 3\, dx \qquad v = \frac{1}{2}e^{2x}$$

$$\int 3x\, e^{2x}\ dx = 3x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x} 3\, dx = \frac{3}{2}xe^{2x} - \frac{3}{2}\int e^{2x}\ dx = \frac{3}{2}xe^{2x} - \frac{3}{2}\left(\frac{1}{2}e^{2x}\right) + C$$

$$= \frac{3}{2}xe^{2x} - \frac{3}{4}e^{2x} + C$$

3. Evaluate the following integral: $\int_{-2}^{1} x\left(x+3\right)^{1/2}\ dx$ **.**

$$\int_{-2}^{1} x\left(x+3\right)^{1/2}\ dx$$

$$u = x \qquad dv = \left(x+3\right)^{1/2}\ dx$$

$$du = dx \qquad v = \frac{2}{3}\left(x+3\right)^{3/2}$$

$$\int_{-2}^{1} x\left(x+3\right)^{1/2}\ dx = x\left[\frac{2}{3}\left(x+3\right)^{3/2}\right]_{-2}^{1} - \int_{-2}^{1}\frac{2}{3}\left(x+3\right)^{3/2}\ dx = \left[\frac{2}{3}x\left(x+3\right)^{3/2}\right]_{-2}^{1} - \frac{2}{3}\left[\frac{2}{5}\left(x+3\right)^{5/2}\right]_{-2}^{1}$$

$$= \left[\frac{2}{3}(1)(1+3)^{3/2}\right] - \left[\frac{2}{3}(-2)(-2+3)^{3/2}\right] - \left[\frac{4}{15}(1+3)^{5/2}\right] + \left[\frac{4}{15}(-2+3)^{5/2}\right]$$

$$= \frac{2}{3}(4)^{3/2} + \frac{4}{3}(1)^{3/2} - \frac{4}{15}(4)^{5/2} + \frac{4}{15}(1)^{5/2} = \frac{2}{3}(8) + \frac{4}{3} - \frac{4}{15}(32) + \frac{4}{15} = \frac{20}{3} - \frac{124}{15} = -\frac{8}{5} = -1.6$$

4. Evaluate the following integral. $\int_{1}^{\infty} \frac{1}{x^{1.5}}\, dx$

NOTE: You must use the correct notation to earn full credit.

$$\int_1^{\infty} \frac{1}{x^{1.5}}\,dx = \lim_{b\to\infty}\int_1^b x^{-1.5}\,dx = \lim_{b\to\infty}\left[\frac{1}{-0.5}x^{-0.5}\right]_1^b = \lim_{b\to\infty}\left[-\frac{2}{x^{0.5}}\right]_1^b = \lim_{b\to\infty}\left[-\frac{2}{b^{0.5}}+\frac{2}{1}\right] = 2$$

5. Consider the $f(x,y)=\frac{4}{\sqrt{xy}}$. What is the domain of this function?

Since anything under a radical must be positive and 0 cannot be in a denominator, xy > 0. this means that both x and y are positive OR they are both negative.

6. Evaluate the following integral. $\int_{-\infty}^{0} x\,e^{-x^2}\,dx$.

NOTE: You must use the correct notation to earn full credit.

$$\int_{-\infty}^{0} x\,e^{-x^2}\,dx = \lim_{a\to-\infty}\int_a^0 x\,e^{-x^2}\,dx = \lim_{a\to-\infty}\left[-\frac{1}{2}e^{-x^2}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{2e^{-x^2}}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{2e^{-0^2}}+\frac{1}{2e^{-a^2}}\right] = -\frac{1}{2}+0 = -\frac{1}{2}$$

NOTE:

$$\int x\,e^{-x^2}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$$

$u = -x^2$

$du = -2x\,dx$

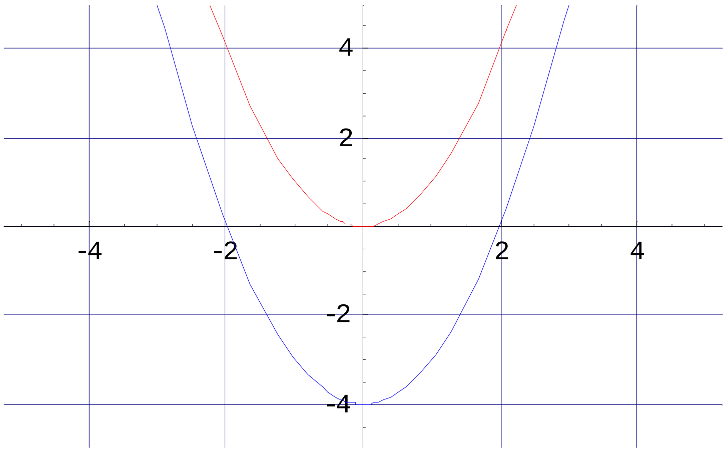
$-\frac{1}{2}\,du = x\,dx$

7. Evaluate the function $f(x,y)=\sqrt{xy+x^2-y^2}$ at each of the following points.

a. (4, 1) $f(4,1)=\sqrt{4(1)+4^2-1^2} = \sqrt{4+16-1} = \sqrt{19}$

b. (-2, 0) $f(-2,0)=\sqrt{(-2)(0)+(-2)^2-0^2} = \sqrt{0+4-0} = \sqrt{4} = 2$

8. On the following set of axes, sketch the level curves for the f(x, y) = $x^2 - y - 1$ for z_0 = -1 and z_0 =3.



Since z = $x^2 - y - 1$, y = $x^2 - 1 - z$.
For z_0 = -1, the level curve is y = x^2 – 1 – (-1) or y = x^2. This is the red curve above.
For z_0 = 3, the level curve is y = x^2 – 1 – 3 or y = x^2 – 4. This is the blue curve.
Both curves are parabolas.

9. Find all four second order partial derivatives of the function $f(x,y)=\sqrt{xy}$. Be sure to label each one correctly.

The easiest way to do it:

$$f(x,y)=x^{1/2}y^{1/2}$$

$$f_x=\frac{1}{2}x^{-1/2}y^{1/2}$$

$$f_{xx}=\frac{1}{2}\left(-\frac{1}{2}x^{-3/2}\right)y^{1/2}=-\frac{1}{4}x^{-3/2}y^{1/2}=-\frac{y^{1/2}}{4x^{3/2}}$$

$$f_{xy}=\left(\frac{1}{2}x^{-1/2}\right)\left(\frac{1}{2}y^{-1/2}\right)=\frac{1}{4}x^{-1/2}y^{-1/2}=\frac{1}{4x^{1/2}y^{1/2}}=\frac{1}{4\sqrt{xy}}$$

$$f_y=x^{1/2}\left(\frac{1}{2}y^{-1/2}\right)$$

$$f_{yy}=x^{1/2}\left(\frac{1}{2}\left(-\frac{1}{2}y^{-3/2}\right)\right)=-\frac{1}{4}x^{1/2}y^{-3/2}=-\frac{1}{4x^{1/2}y^{3/2}}$$

$$f_{yx}=\left(\frac{1}{2}x^{-1/2}\right)\left(\frac{1}{2}y^{-1/2}\right)=\frac{1}{4}x^{-1/2}y^{-1/2}=\frac{1}{4x^{1/2}y^{1/2}}=\frac{1}{4\sqrt{xy}}$$

10. Find all three first order partial derivatives of the function f(x, y, z) = ln(2x + 3y – 4z). Be sure to label each one correctly.

Remember: $\frac{d}{dx}[\ln u]=\frac{u'}{u}=\frac{\frac{du}{dx}}{u}$

$$f(x,y,z)=\ln(2x+3y-4z)$$

$$f_x=\frac{2}{2x+3y-4z}$$

$$f_y=\frac{3}{2x+3y-4z}$$

$$f_z=\frac{-4}{2x+3y-4z}$$

SECOND FORM

1. Evaluate the following integral: $\int x^3\ln x dx$.

$$\int x^3\ln x dx$$

$u=\ln x$ $\qquad dv=x^3\ dx$

$du=\frac{1}{x}dx$ $\qquad v=\frac{1}{4}x^4$

$$\int x^3\ln x dx=\ln x\left(\frac{1}{4}x^4\right)-\int\left(\frac{1}{4}x^4\right)\frac{1}{x}dx=\frac{1}{4}x^4\ln x-\frac{1}{4}\int x^3\ dx$$

$$=\frac{1}{4}x^4\ln x-\frac{1}{4}\left(\frac{1}{4}x^4\right)+C=\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+C$$

2. Evaluate the following integral: $\int 4x\, e^{5x}\ dx$**.**

$\int 4x\, e^{5x}\ dx$

$u = 4x \qquad dv = e^{5x}\ dx$

$du = 4\ dx \qquad v = \frac{1}{5}e^{5x}$

$$\int 4x\, e^{5x}\ dx = 4x\left(\frac{1}{5}e^{5x}\right) - \frac{4}{5}\int e^{5x}\ dx = \frac{4}{5}e^{5x} - \frac{4}{5}\left(\frac{1}{5}e^{5x}\right) + C = \frac{4}{5}e^{5x} - \frac{4}{25}e^{5x} + C$$

3. Evaluate the following integral: $\int_{-4}^{4} x\left(x+5\right)^{1/2}\ dx$**.**

$\int_{-4}^{4} x\left(x+5\right)^{1/2}\ dx$

$u = x \qquad dv = \left(x+5\right)^{1/2}\ dx$

$du = dx \qquad v = \frac{2}{3}\left(x+5\right)^{3/2}$

$$\int_{-4}^{4} x\left(x+5\right)^{1/2}\ dx = \left[\frac{2}{3}x\left(x+5\right)^{3/2}\right]_{-4}^{4} - \int_{-4}^{4}\frac{2}{3}\left(x+5\right)^{3/2}\ dx = \left[\frac{2}{3}x\left(x+5\right)^{3/2}\right]_{-4}^{4} - \left[\frac{2}{3}\left(\frac{4}{5}\right)\left(x+5\right)^{5/2}\right]_{-4}^{4}$$

$$= \frac{2}{3}(4)(4+5)^{3/2} - \frac{2}{3}(-4)(-4+5)^{3/2} - \frac{4}{15}(4+5)^{5/2} + \frac{4}{15}(-4+5)^{5/2}$$

$$= \frac{8}{3}\left(9^{3/2}\right) + \frac{8}{3}\left(1^{3/2}\right) - \frac{4}{15}\left(9^{5/2}\right) + \frac{4}{15}\left(1^{5/2}\right) = \frac{8}{3}(27) + \frac{8}{3} - \frac{4}{15}(243) + \frac{4}{15}$$

$$= \frac{216}{3} + \frac{8}{3} - \frac{972}{15} + \frac{4}{15} = \frac{152}{15} = 10.1333$$

4. Evaluate the following integral. $\int_{1}^{\infty}\frac{1}{x^{2.5}}\ dx$

NOTE: You must use the correct notation to earn full credit.

$$\int_{1}^{\infty}\frac{1}{x^{2.5}}\ dx = \lim_{b\to\infty}\int_{1}^{b} x^{-2.5}\ dx = \lim_{b\to\infty}\left[\frac{1}{-1.5}x^{-1.5}\right]_{1}^{b} = \lim_{b\to\infty}\left[-\frac{2}{3x^{1.5}}\right]_{1}^{b} = \lim_{b\to\infty}\left[-\frac{2}{3b^{1.5}} + \frac{2}{1^{1.5}}\right] = 0 + 2 = 2$$

5. Consider the $f\left(x,y\right) = \frac{5}{\sqrt{3xy}}$**. What is the domain of this function?**

Since anything under a radical must be positive and 0 cannot be in a denominator, xy > 0. this means that both x and y are positive OR they are both negative.

6. Evaluate the following integral. $\int_{-\infty}^{0} x\, e^{-2x^2}\ dx$**.**

NOTE: You must use the correct notation to earn full credit.

$$\int_{-\infty}^{0} x\, e^{-2x^2}\, dx = \lim_{a\to-\infty}\int_{a}^{0} x\, e^{-x^2}\, dx = \lim_{a\to-\infty}\left[-\frac{1}{4}e^{-2x^2}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{4e^{-2x^2}}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{4e^{-0^2}}+\frac{1}{4e^{-a^2}}\right] = -\frac{1}{4}+0 = -\frac{1}{4}$$

NOTE:

$$\int x\, e^{-2x^2}\, dx = -\frac{1}{4}\int e^{u}\, du = -\frac{1}{4}e^{u} + C = -\frac{1}{4}e^{-2x^2} + C$$

$u = -2x^2$

$du = -4x\, dx$

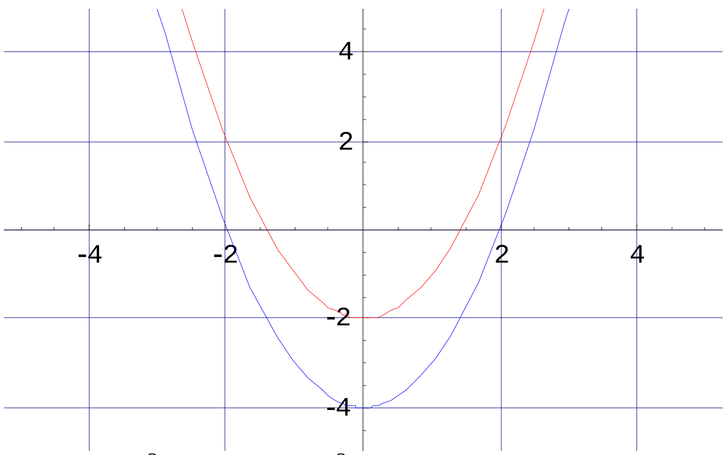
$-\frac{1}{4}\, du = x\, dx$

7. Evaluate the function $f(x,y) = \sqrt{xy - x^2 + y^2}$ **at each of the following points.**

a. (5, 2) $f(5,2) = \sqrt{5(2) - 5^2 + 2^2} = \sqrt{10 - 25 + 4} = \sqrt{-11}$

b. (2, -1) $f(2,-1) = \sqrt{2(-1) - 2^2 + (-1)^2} = \sqrt{-2 - 4 + 1} = \sqrt{-5}$

8. On the following set of axes, sketch the level curves for the f(x, y) = $x^2 - y - 2$ for $z_0 = 0$ and $z_0 = 2$.



Since z = $x^2 - y - 2$, y = $x^2 - 2 - z$.
For $z_0 = 0$, the level curve is y = $x^2 - 2 - 0$ or y = $x^2 - 2$. This is the red curve above.
For $z_0 = 2$, the level curve is y = $x^2 - 2 - 2$ or y = $x^2 - 4$. This is the blue curve.
Both curves are parabolas.

9. Find all four second order partial derivatives of the function $f(x,y) = \sqrt{3xy}$. **Be sure to label each one correctly**.

The easiest way to do it:

$$f\left(x, y\right) = \sqrt{3}x^{1/2}y^{1/2}$$

$$f_x = \frac{\sqrt{3}}{2}x^{-1/2}y^{1/2}$$

$$f_{xx} = \frac{\sqrt{3}}{2}\left(-\frac{1}{2}x^{-3/2}\right)y^{1/2} = -\frac{\sqrt{3}}{4}x^{-3/2}y^{1/2} = -\frac{\sqrt{3}y^{1/2}}{4x^{3/2}}$$

$$f_{xy} = \left(\frac{\sqrt{3}}{2}x^{-1/2}\right)\left(\frac{1}{2}y^{-1/2}\right) = \frac{\sqrt{3}}{4}x^{-1/2}y^{-1/2} = \frac{\sqrt{3}}{4x^{1/2}y^{1/2}} = \frac{\sqrt{3}}{4\sqrt{xy}}$$

$$f_y = \sqrt{3}x^{1/2}\left(\frac{1}{2}y^{-1/2}\right)$$

$$f_{yy} = \sqrt{3}x^{1/2}\left(\frac{1}{2}\left(-\frac{1}{2}y^{-3/2}\right)\right) = -\frac{\sqrt{3}}{4}x^{1/2}y^{-3/2} = -\frac{\sqrt{3}}{4x^{1/2}y^{3/2}}$$

$$f_{yx} = \sqrt{3}\left(\frac{1}{2}x^{-1/2}\right)\left(\frac{1}{2}y^{-1/2}\right) = \frac{\sqrt{3}}{4}x^{-1/2}y^{-1/2} = \frac{\sqrt{3}}{4x^{1/2}y^{1/2}} = \frac{\sqrt{3}}{4\sqrt{xy}}$$

10. Find all three first order partial derivatives of the function f(x, y, z) = ln(4x + 6y – 3z). Be sure to label each one correctly.

Remember: $\frac{d}{dx}\left[\ln u\right] = \frac{u'}{u} = \frac{\frac{du}{dx}}{u}$

$$f\left(x, y, z\right) = \ln\left(4x + 6y - 3z\right)$$

$$f_x = \frac{4}{4x + 6y - 3z}$$

$$f_y = \frac{6}{4x + 6y - 3z}$$

$$f_z = \frac{-3}{4x + 6y - 3z}$$