

There are a total of 7 problems. Please be sure that you have the entire test. Show all necessary work. Answers that seem to appear by magic will receive no credit.

Your work needs to be done on separate paper (or electronic file) and must be neat and easy to read. I suggest that you work out your solutions and then copy them to the paper you plan to submit. If you submit your answers using Mathematica, each problem must be clearly labeled. All solutions must be submitted in numerical order.

You are to do your own work. You may use any books, your class notes, and Mathematica. This is due at the **BEGINNING** of class on **Wednesday, February 21**; late papers will be heavily penalized. If you wish, you may email your solutions. In that case, your work must be sent **BEFORE** coming to class.

Each problem is worth 7 points.

1. Use Mathematica to graph the slope field for the differential equation $\frac{dy}{dx} = \sin x \cos 2x$.

Also, place the solution curve which corresponds to the initial value $y(-2) = -1$.

2. A tank holds 500 gallons of liquid. Initially, it contains 100 gallons which has 10 pounds of a salt dissolved in it. Brine, which contains 1 pound of salt per 10 gallons of liquid, pours into the tank at the rate of 5 gallons per second. The well-mixed solution drains from the tank at the rate of 3 gallons per second. How much salt is in the tank at the moment that the tank starts to overflow?

(Note: This requires finding and solving a differential equation that determines the amount of salt in the tank at any time t . Then the solution must be evaluated for the value of t which corresponds to when the tank overflows.)

3. Solve the differential equation $(3 + y \cos(xy)) dx + (x \cos(xy) - 8y) dy = 0$

4. Solve the differential equation $x^2 y'' + 3xy' = 2$.

5. Solve the initial value problem

$$y^{(4)} + y^{(3)} - 2y'' + 4y' - 24y = 0; y(0) = 11, y'(0) = -9, y''(0) = 53, y'''(0) = -95$$

6. Use the Method of Undetermined Coefficients to solve the differential equation

$$y'''' + 2y'' - 3y' - 10y = 8xe^{-2x}.$$

7. Use the Method of Variation of Parameters to solve the differential equation

$$y'' + y = \frac{1}{1 + \sin x}.$$