NAME: *Solutions*

Math 20 E Practice Test III for 4/11/08

SHOW EACH STEP IN YOUR SOLUTIONS, AND PRESENT YOUR ANSWERS CLEARLY.

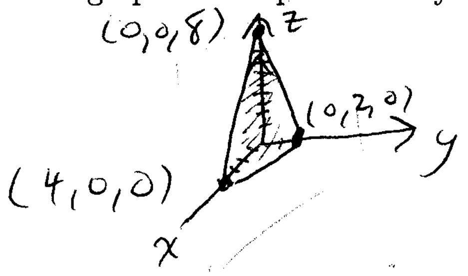
1. In this problem, $f(x,y) = 8 - 2x - 4y$.

   a. How can you tell that the graph of $f(x,y)$ will be a plane? *linear function*

   b. Give the coordinates of three different points on this plane.

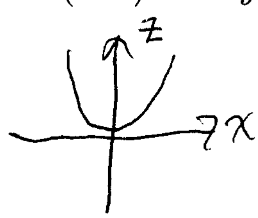
   $(0,0,8) \quad (0,2,0) \quad (4,0,0)$

   c. Sketch the graph of this plane with your three points clearly shown.

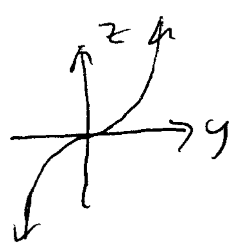


2. In this problem $f(x,y) = x^2 + y^3$.

   a. Sketch the cross section (slice) when $y = 0$.



   b. Sketch the cross section (slice) when $x = 0$.



   c. Based on your previous answers does $f(x,y)$ have a maximum, minimum, or neither at $(0,0)$?

   Neither, seen in 2b.

3. Use the H-test to find the two critical points of $f(x,y) = -x^3 - y^2 + 3x - 2y$ and identify what type they are.

$f_x = -3x^2 + 3 \qquad f_y = -2y - 2 \qquad (\pm 1, 1)$

$f_{xx} = -6x \qquad f_{yx} = 0$

$f_{xy} = 0 \qquad f_{yy} = -2$

$H = (-6x)(-2) - (0)^2 = 12x$

at $(1,1)$ $H = 12 \Rightarrow$ relative max or min, $f_{xx} = -6 \Rightarrow$ relative max

at $(-1,1)$ $H = -12 \Rightarrow$ saddle

4. A digital thermometer manufacturer has developed the monthly profit equation $P(x,y) = -2x^2+4xy-3y^2+4x-2y+77$ where x is the number (in hundreds) of indoor thermometers and y is the number (in hundreds) of indoor/outdoor thermometers. If $P(x,y)$ is in thousands of dollars, find the maximum monthly profit.

$\frac{\partial P}{\partial x} = -4x+4y+4$ $\quad \frac{\partial P}{\partial y} = 4x-2\cdot 6y \Rightarrow$ Critical pts when both $=0$ $\Rightarrow$ add to get $-2y+2=0$, $y=1$ $\Rightarrow$ substitute $y=1$ get $x=2$

$\frac{\partial^2 P}{\partial x^2} = -4$ $\quad \frac{\partial^2 P}{\partial x \partial y} = 4$ $\quad H=(-4)(-6)-4^2 = 24-16 = 8>0 \Rightarrow$ rel. max. or min.

$\frac{\partial^2 P}{\partial y \partial x} = 4$ $\quad \frac{\partial^2 P}{\partial y^2} = -6$ $\quad \frac{\partial^2 P}{\partial x^2} = -4<0 \Rightarrow$ rel. max.

So max at $(x,y)=(2,1)$ of $P(2,1) = \$80$ thousand

5a. Use Lagrange multipliers to find the value of x and y which maximize the function $f(x,y) = x^2-y^2$ subject to the constraint $2x+y-3=0$. $\quad g(x,y) = 2x+y-3$

$f_x = 2x \quad f_y = -2y$ $\quad$ substitute in constraint & get

$g_x = 2 \quad g_y = 1$ $\quad 2(-2y)+y-3=0 \Rightarrow -3y=3$

$\lambda = \frac{2x}{2} = \frac{-2y}{1}$ so $x=-2y$ $\quad$ So $y=-1$

Then $x=-2y=2$

So $(x,y)=(2,-1)$. Max. Value is $f(2,-1) = 2^2-(-1)^2 = 3$

5b. Solve the same problem using the substitution method.

$2x+y-3=0$ gives $y=3-2x$. Substituting in x^2-y^2 gives

$x^2-(3-2x)^2 = x^2-(9-12x+4x^2) = -3x^2+12x-9$.

Maximize this by finding critical pt when derivative is 0: $-6x+12=0$

So $12=6x$ and $\underline{x=2}$, From $y=3-2x$ we get $\underline{y=-1}$. Second deriv $-6<0 \Rightarrow$ rel. max.

6. Evaluate the double integral $\int_0^2 \left(\int_x^1 12xy\,dy \right) dx$

$$\int_x^1 12xy\,dy = 6xy^2\Big|_{y=x}^{y=1} = 6x-6x(x^2) = 6x-6x^3$$

$$\int_0^2 (6x-6x^3)\,dx = 3x^2-\frac{3x^4}{2}\Big|_0^2 = 12-24 = \boxed{-12}$$