Math 333 Homework Assignment 2
J. Sands Due 9/29/09

Remember that collaboration is encouraged, but you must write up solutions in your own words. Style and clarity of exposition are important elements to be considered in your solutions.

1. page 102, number 2.

2. page 102, number 3.

3. page 102, number 7.

4. page 109, number 6.

5. page 109, number 8.

6. page 110, number 10.

7. Example 15.5, do what is left to the reader.

8. page 119, number 12.

9. page 119, number 15.

10. Let $X \neq \emptyset$ be a set and $\sim$ be an equivalence relation on X with the property that every equivalence class is finite. Let S be the collection of all subsets A of X such that A is a (possibly empty) union of equivalence classes.

a) Show that $S = \{A \subset X : a \in A \text{ and } b \sim a \implies b \in A\}$.

b) Prove that S is a σ-algebra.

Define $\mu : S \to [0, \infty]$ by $\mu(A) = |A|$. (Put $\mu(A) = \infty$ if A is infinite.)

c) Show that μ is a measure on S.

d) Show that if $B \subset X$, then $\mu^*(B) = |\overline{B}|$, where $\overline{B} = \{x \in B : x \sim b \text{ for some } b \in B\}$.

e) Prove that for the outer measure μ^*, the collection of measurable sets is $\Lambda = S$.