Math 52 Homework Assignment 4
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As always, style and clarity of exposition are important elements to be considered in your solutions.

Reminder: Our first test will be in class on Wednesday, October 14
The test will cover Chapter 1, sections 1,2,4,5,6, and the supplementary material on sets and geometric series. You should know how to do standard types of problems such as in the homework, without referring to your text or notes. You should also be familiar with the definitions and theorems that we have studied.

1. Write down all subsets of $\{a, b, c, d\}$

2. Let $S = \{x \in \mathbb{R} : x > 7\}$ and $T = \{x \in \mathbb{R} : x^3 > 100\}$. Prove that $S \subset T$.

3. Now let $S = \{x \in \mathbb{R} : x > 7\}$ and $W = \{x \in \mathbb{R} : x^3 > 343\}$. Prove that $S = W$.

4. Suppose A and B are subsets of a set U (sometimes called the universal set). Let A^c denote the complement of A in U and similarly let B^c denote the complement of B in U. The definition of A^c is $A^c = \{x \in U : x \notin A\}$. Show that the complement of $A \cup B$ is $A^c \cap B^c$, that is: $(A \cup B)^c = A^c \cap B^c$. This is also referred to as one of DeMorgan's laws, so that may give you some idea of what you need to use in order to prove it.

5. Suppose that $S = \{x \in X : P(x)\}$ and $T = \{x \in X : Q(x)\}$. Show that $S \cap T = \{x \in X : P(x) \wedge Q(x)\}$.

6. Suppose that A, B, and C are subsets of U Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by showing that $x \in A \cap (B \cup C)$ if and only if $x \in (A \cap B) \cup (A \cap C)$, and using the definitions of union and intersection.

7. Let $A = \{n \in \mathbb{Z}^+ : n|42\}$ and $B = \{n \in \mathbb{Z}^+ : n|70\}$.
a. List the elements of A.
b. List the elements of B.
c. List the elements of $A \cup B$.
d. List the elements of $A \cap B$.
e. Show that $|A| + |B| = |A \cap B| + |A \cup B|$.
f. (Extra credit) Show that the equation in part (e) is true for any two finite sets A and B.

8. Let $S = \{n^2 : n \in \mathbb{Z}^+\}$ and $T = \{m \in \mathbb{Z}^+ : \sqrt{m} \in \mathbb{Z}\}$ Show that $S = T$.

9a. Write in set notation"W is the set of all real solutions to the equation $x^4 - 4x^2 + 3 = 0$".
b. Show that $W = \{1, -1, \sqrt{3}, -\sqrt{3}\}$