SAMPLE PROOF BY INDUCTION
Math 52C Fall 2009
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You want to prove that a certain statement is true for all positive integers **n** greater than or equal to some integer **a**. Identify what that statement is before writing the proof. We will call it P(n). Also identify the starting value **a**. The proof consists of showing that P(a) is true and that P(N) implies P(N+1) for each $N \geq a$.

Theorem: The statement that $10^n+3(4^{n+2})+5$ is divisible by 9 is true for all integers $n\geq 0$.

Proof: The proof is by induction on n.

P(n) will be the statement $10^n+3(4^{n+2})+5=9k$ for some integer k.

1) **Basis Step: We prove P(0), which says $10^0+3(4^{0+2})+5$ is divisible by 9**
   Now $10^0+3(4^{0+2})+5= 1+3(4^2)+5=1+3(16)+5=1+48+5=54$. Since 54=9(6), we see that $10^0+3(4^{0+2})+5$ is divisible by 9, so P(0) is true.

2) **Inductive Step: The inductive hypothesis is P(N), which says $10^N+3(4^{N+2})+5=9k$ for some integer k. We will use this to prove P(N+1), which says $10^{N+1}+3(4^{N+1+2})+5=9j$ for some integer j.**

   Our assumption is that $10^N+3(4^{N+2})+5=9k$, which may be rewritten as $10^N=9k-3(4^{N+2})-5$. Starting with the left hand side of the equation in P(N+1), we will show by a sequence of steps that it is equal to the right hand side.
   By rules for exponents, $10^{N+1}+3(4^{N+1+2})+5=10(10^N)+3(4^{N+1+2})+5$
   By the inductive hypothesis, we may substitute in $10^N=9k-3(4^{N+2})-5$, to find that this equals $10(9k-3(4^{N+2})-5)+3(4^{N+1+2})+5$, which simplifies algebraically to $90k-30(4^{N+2})-50+3(4)(4^{N+2})+5=90k-30(4^{N+2})+12(4^{N+2})-45=$ $90k+(-30+12)(4^{N+2})-45=90k-18(4^{N+2})-45=9(10k-2(4^{N+2})-5)=9j$. Combining these equalities shows that **$10^{N+1}+3(4^{N+1+2})+5=9j$ for some integer j.**

   We have proved P(N+1) under the assumption of P(N). By the Principal of Mathematical Induction, we conclude that P(n) is true for all integers $n\geq 0$.