TEMPLATE FOR PROOF BY INDUCTION
Math 52C Fall 2009

You want to prove that a certain statement is true for all positive integers **n** greater than or equal to some integer **a**. Identify what that statement is before writing the proof. We will call it P(n). Also identify the starting value **a**. The proof consists of showing that P(a) is true and that P(N) implies P(N+1) for each N ≥a.

Theorem: The statement …… is true for all positive integers n≥a.
(Write out the statement P(n).)

Proof: The proof is by induction on n.

P(n) will be the statement ……
(Write out the statement P(n) again.)

1) **Basis Step: We prove P(a), which says**
   ……
   (Write out what you get when you substitute n=a into the statement P(n).)
   Now show that this statement is true. It is usually very easy!

2) **Inductive Step: The inductive hypothesis is P(N) , which says**
   ……
   (Write out what you get when you substitute n=N into the statement P(n).)
   We will use this to prove P(N+1), which says
   ……..……..
   (Carefully write out what you get when you substitute n=N+1 into the statement P(n). One way to do this is to put parentheses around each n, and then replace each n by N+1 inside the parentheses.)

   (Now look at the statement P(N+1) that you have just written and consider how to prove it. Start with one of the expressions in P(N+1) and rewrite it so that a part of it perfectly matches what is described by P(N). Give a valid reason behind each step that uses known facts as you do this. You may use the fact that N ≥a.)
   By the inductive hypothesis, we obtain ……
   (Write out what results when you use P(N) to rewrite the part that you have matched.)

   (You should now be very close to arriving at the conclusion expressed in P(N+1). Add more steps using known facts until you can reach this conclusion.)
   We have proved P(N+1) under the assumption of P(N). By the Principal of Mathematical Induction, we conclude that P(n) is true for all n≥a.