

NAME:

Math 52C Practice for Test I October 14, 2009

INTEGERS

1a. Use the Euclidean algorithm to find $\text{GCD}(255,68)$.

b. Are 255 and 68 relatively prime?

GEOMETRIC SERIES: PROVING AND DISPROVING

2a. Write the negation of the following statement in a way that changes the quantifier. “There exists a positive integer n such that $6^n - 1$ is not a multiple of 5.

b. Prove the statement that you have just written in part a.

c. Write the negation of the following statement in a way that changes the quantifier. “For all positive integers n , the integer $6^n - 2$ is a multiple of 4.

d. Prove the statement that you have just written in part c.

LOGIC

3a. Prove that $P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$. What is this rule called?

3b. Show that $P \wedge (Q \vee P)$ is equivalent to P .

4a. Write down two different ways of expressing the negation of the following statement about a real number b . “Either $b > 2$ or $b < -2$.”

b. Write down the converse of the following statement. “If the polynomial $x^2 + bx + 1$ has two real roots then $b > 2$ or $b < -2$.”

c. Write down the contrapositive of the following (same) statement. “If the polynomial $x^2 + bx + 1$ has two real roots then $b > 2$ or $b < -2$.”

d. Prove that the polynomial $x^2 + bx + 1$ has two real roots if and only if $b > 2$ or $b < -2$. Notice that this can be done by proving the statement you have written down in part (b) and the statement you have written down in part (c). You may use the quadratic formula.

SETS

5. Define $S = \{n \in \mathbb{N} : 30|n\}$, $T = \{n \in \mathbb{N} : 5|n\}$ and $W = \{n \in \mathbb{N} : 3|n\}$.

a. Show that $S \subset T$.

b. Show that $S \subset W$.

c. Show that $S \subset (T \cap W)$.

d. Show that $S \neq (T \cap W)$.

e. Show that S is closed under addition. What rules of arithmetic are used?

f. Show that S is not closed under division.

RELATIONS

6. Suppose $S = \mathbb{N}$, and $\rho = \{(s, t) \in S \times S : s \pmod{5} = t \pmod{5}\}$.

a. Show that ρ is an equivalence relation.

b. Write down the equivalence class of 17 for this relation.

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c. Write down the equivalence class of 15 for this relation.

d. We define a binary relation on \mathbb{N} by $\sigma = \{(a, b) : a|b\}$. Show that σ is reflexive and transitive, but not symmetric.