

NAME:

**Math 52C Practice for Test II on Wednesday, November 11, 2009**

Explain each step along the way in order to fully demonstrate your knowledge

**1. FUNCTIONS**

- a. According to our formal definition, what is a function?
  - b. Let  $S = \{2, 4, 6\}$  and  $T = \{7, 8, 9, 10\}$ . Define a relation  $\sigma$  from  $S$  to  $T$  by  $\sigma = \{(2, 8), (4, 7), (6, 9)\}$ . Show that  $\sigma$  is a function.
  - c. What is the definition of one-to-one?
  - d. Show that  $\sigma$  is one-to-one.
  - e. Why doesn't  $\sigma$  have an inverse function from  $T$  to  $S$ ?
  - f. Now suppose that we consider  $\sigma$  to be a function from  $S = \{2, 4, 6\}$  to  $R = \{7, 8, 9\}$ . Write down the inverse function of  $\sigma$  from  $R$  to  $S$ .
  - g. Suppose that  $f$  and  $g$  are functions for which we can define the composition  $f \circ g$ . If  $f$  and  $g$  are one-to-one, prove that  $f \circ g$  is one-to-one.
  - h. If  $f$  and  $g$  have inverse functions, prove that  $f \circ g$  has an inverse, namely  $g^{-1} \circ f^{-1}$ .
- Suppose that  $X = \mathbb{R}^+$  is the set of positive real numbers and  $Y = \mathbb{N}$  is the set of positive integers, and  $f$  is the function from  $\mathbb{R}^+$  to  $\mathbb{N}$  that rounds a real number up to the closest integer greater than or equal to it. So  $f(42.1) = 43$ ,  $f(7.9) = 8$ ,  $f(10) = 10$ , etc.
- i. Is  $f$  one-to-one? Explain.
  - j. Show that  $f$  is onto.
  - k. Does  $f$  have an inverse function? Explain.

**2. COMPLETE INDUCTION AND RECURSION**

- a. A sequence is defined by setting  $a_1 = 4$ ,  $a_2 = 10$ , and  $a_{n+1} = 4a_n - 3a_{n-1}$  for each integer  $n \geq 2$ . Show by complete induction that  $a_n = 3^n + 1$  for all positive integers  $n$ .
- b. Find a formula for  $b_n$  if  $b_1 = 6$ ,  $b_2 = 18$ , and  $b_{n+1} = 4b_n - 3b_{n-1}$  for each integer  $n \geq 2$ .

**3. EXTENDED EUCLIDEAN ALGORITHM**

- a. Use the Euclidean Algorithm to find integer values of  $x$  and  $y$  such that  $86x + 38y = 2$ . Show your work.
- b. What is the GCD of 86 and 38?
- c. Find values of  $z$  and  $w$  so that  $86x + 38y = 6$
- d. Can you find values of  $s$  and  $t$  so that  $86s + 38t = 7$ ? Why or why not?
- e. State the Fundamental Theorem of Arithmetic
- f. Suppose that  $b \mid c$  and  $a \mid c$  and  $\text{GCD}(a,b)=1$ . Show that  $ab \mid c$ .