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Math 52 Test I February 27, 2009

INTEGERS

1a. Use the Euclidean algorithm to find GCD(255,68). $= \boxed{17}$

$$(255) = 3(68) + (51)$$
$$(68) = 1(51) + (17)$$
$$(51) = 3\cdot(17) + (0)$$

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b. Are 255 and 68 relatively prime? No $GCD(255,68) \neq 1$.

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NUMBER REPRESENTATIONS

2a. Convert $(.13)_5$ to base 10.

$$1 \times \frac{1}{5} + 3 \times \frac{1}{5^2} = \frac{5+3}{25} = \boxed{\frac{8}{25}} \quad (= .32).$$

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b. Convert $(.131313...)_5$ to base 10.

$$\frac{8}{25} \times \frac{1}{1-\frac{1}{5^2}} = \boxed{\frac{8}{24} = \frac{1}{3}} \quad (= .333...)$$

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c. Convert 113 to base 5.

113 mod 5 = 3

22 mod 5 = 2

4 mod 5 = 4

$\boxed{(423)_5}$

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LOGIC

3a. Construct the truth table for $P \wedge (Q \vee \neg P)$.

P	Q	$\neg P$	$Q \vee \neg P$	$P \wedge (Q \vee \neg P)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

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3b. Show that $P \wedge (Q \vee \neg P)$ is equivalent to $P \wedge Q$.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Same as above last column.

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4a. Write down two different ways of expressing the negation of the statement: "Either $b > 2$ or $b < -2$."

It is not the case that $b > 2$ or $b < -2$.

$b \leq 2$ and $b \geq -2$

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4b. Write down the converse of the following statement about a real number b. "If $b > 2$ or $b < -2$ then the polynomial $x^2 + bx + 1$ has two real roots."

If the polynomial $x^2 + bx + 1$ has two real roots then $b > 2$ or $b < -2$.

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c. Write down the contrapositive of the following (same) statement. "If $b > 2$ or $b < -2$ then the polynomial $x^2 + bx + 1$ has two real roots."

If the polynomial $x^2 + bx + 1$ does not have two real roots, then $b \leq +2$ and $b \geq -2$.

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SETS

5. Define $S = \{n \in \mathbb{N} : 30|n\}$, $T = \{n \in \mathbb{N} : 5|n\}$ and $W = \{n \in \mathbb{N} : 3|n\}$.

   a. Show that $S \subset T$.

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$s \in S \Rightarrow 30|s \Rightarrow s = 30k \Rightarrow s = 5(6k) \Rightarrow 5|s$
$\Rightarrow s \in T$. We have shown that $s \in S \Rightarrow s \in T$.
This is exactly what it means to say that $S \subset T$.

   b. Show that $S \subset W$.

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$s \in S \Rightarrow 30|s \Rightarrow s = 30k \Rightarrow s = 3(10k) \Rightarrow 3|s$
$\Rightarrow s \in W$. We have shown that $s \in S \Rightarrow s \in W$.
Thus $S \subset W$.

   c. Show that $S \subset (T \cap W)$.

From above $s \in S \Rightarrow s \in T \wedge s \in W \Rightarrow s \in T \cap W$ (def of $\cap$)

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So $S \subset T \cap W$

   d. Show that $S \neq (T \cap W)$.

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$15 \in T \cap W$
but $15 \notin S$
So $S \neq T \cap W$

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RELATIONS

6. Suppose $S = I\!N$, and $\rho = \{(s,t) \in S \times S : s \pmod 5 = t \pmod 5\}$.

   a. Show that ρ is an equivalence relation. We show reflexive, symmetric, transitive.

   $s \bmod 5 = s \bmod 5 \Rightarrow (s,s) \in \rho \Rightarrow$ reflexive

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   $(s,t) \in \rho \Rightarrow s \bmod 5 = t \bmod 5 \Rightarrow t \bmod 5 = s \bmod 5$
   $\Rightarrow (t,s) \in \rho$ so symmetric

   $(s,t) \in \rho$ and $(t,u) \in \rho \Rightarrow s \bmod 5 = t \bmod 5$ and
   $t \bmod 5 = u \bmod 5 \Rightarrow s \bmod 5 = u \bmod 5 \Rightarrow (s,u) \in \rho$
   so transitive

   b. Write down the equivalence class of 17 for this relation.

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   $[17] = \{2, 7, 12, 17, 22, 27, \ldots\}$

FUNCTIONS

7. Suppose that $X = \mathbb{R}^+$ is the set of positive real numbers and $Y = I\!N$ is the set of positive integers, and f is the function from $\mathbb{R}^+$ to $I\!N$ that rounds a real number up to the closest integer greater than or equal to it. So $f(42.1) = 43$, $f(7.9) = 8$, $f(10) = 10$, etc.

   a. Is f one-to-one? Explain.

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   No, $f(1.5) = 2 = f(1.6)$

   b. Show that f is onto. If $n \in I\!N$, $f(n) = n$, so

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   every $n \in I\!N$ is an output of f. This means f is onto.

   c. Does f have an inverse function? Explain.

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   No, because it is not ~~~~ one-to-one.

   d. What special properties does a relation ρ between X and Y need to have in order to be a function from X to Y?

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   For each (input) $x \in X$, there must be one and only one y such that $(x,y) \in f$.

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