

NAME:

Math 52 Test I October 15, 2003

INTEGERS

1a. Use the Euclidean algorithm to find $\text{GCD}(275,110)$.

b. Are 275 and 110 relatively prime?

NUMBER REPRESENTATIONS

2a. Convert 1202_3 to base 10.

b. Convert 22 to base 2.

LOGIC

3a. Construct the truth table for $P \vee (Q \wedge P)$.

3b. Show that $P \vee (Q \wedge P)$ is equivalent to P .

4a. Write down two different ways of expressing the negation of the statement: “ $a < 2$ and $a > 0$.”

4b. Write down the converse of the following statement about a real number a . “If $a < 2$ and $a > 0$ then $a^2 - 2a < 0$.”

c. Write down the contrapositive of the following (same) statement. “If $a < 2$ and $a > 0$ then $a^2 - 2a < 0$.”

SETS

5. Suppose that A , B , and C are all subsets of a set X . Supply a reason behind each step in the following proof that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

$$x \in A \cup (B \cap C) \iff (x \in A) \vee (x \in B \cap C) \dots\dots$$

$$\iff (x \in A) \vee (x \in B \wedge x \in C) \dots\dots$$

$$\iff (x \in A \vee x \in B) \wedge (x \in A \vee x \in C) \dots\dots$$

$$\iff (x \in A \cup B) \wedge (x \in A \cup C) \dots\dots$$

$$\iff x \in (A \cup B) \cap (A \cup C) \dots\dots$$

$$\text{Thus } x \in A \cup (B \cap C) \iff x \in (A \cup B) \cap (A \cup C) \dots\dots$$

We conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \dots\dots$

RELATIONS

6. Suppose $S = \mathbb{N}$, and ρ is the binary relation on S defined by $\rho = \{(x, y) : x \bmod 10 = y \bmod 10\}$.
- Show that ρ is an equivalence relation.

b. Write the equivalence class of 23 for this equivalence relation.

c. Give an example of a partial order relation on \mathbb{R} .

FUNCTIONS

7. We define the function $f : \mathbb{N} \rightarrow \mathbb{N}$ by $f(n) = 2n$.

a. Is f one-to-one? Explain.

b. Show that f is not onto.

c. Does f have an inverse function? Explain.