Anagrams and Factorials

Lecture Notes for CS 32

Delivered on September 10, 2007

Anagrams: Review

In CS 32, the word *anagram* signifies a permutation, or rearrangement of a group of letters or symbols. Thus the letters in the word *CAT* has *six* different anagrams:

ACT	ATC
CAT	CTA
TAC	TCA

(Note that only two of these are English words, the remaining four are jibberish, but are nevertheless anagrams, by our definition.)

The number of different anagrams of a word having n different letters equals

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1.$$

The notation $n!$ is called "n-factorial."

Anagrams: Review

If the initial word has repeated letters, then the number of anagrams is reduced. Thus the word *MALL* has *twelve* anagrams:

ALLM	ALML	AMLL			
LALM	LAML	LLAM	LLMA	LMAL	LMLA
MALL	MLAL	MLLA			

With four letters, we might expect to find $4!$ permuations. However this number *overcounts* the correct value, because *LL* has only one anagram, not two. Thus we need to divide $4!$ by $2!$. Thus

$$\frac{4!}{2!} = \frac{24}{2} = 12.$$

Anagrams: Review

How many anagrams are in the word *LULL*?

How many are in *TOOT*?

Anagrams: Review

Recall, that a word that consists of k different letters of the alphabet, with r_1 repetitions of the first letter, r_2 of the second, ..., r_k of the k-th letter; the number of unique permutations equals

$$\frac{(r_1 + r_2 + \cdots + r_k)!}{r_1!\ r_2!\ \cdots\ r_k!}.$$

Thus the number of anagrams of the 11 letter word *ABRACADABRA*, which has 5 *A*s, 2 *B*s, 1 *C*, 1 *D*, and 2 *R*s, equals

$$\begin{aligned}\frac{11!}{5!\ \cdot 2!\ \cdot 1!\ \cdot 1!\ \cdot 2!} &= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \cdot 2 \cdot 2} \\ &= 11 \times 10 \times 9 \times 2 \times 7 \times 6 \\ &= 99 \times 840 = 83,160.\end{aligned}$$

Whenever one has a fraction with factorials appearing in both the numerator and denominator, one can always cancel common factors.

Computing Factorials

Whenever one has a fraction with factorials appearing in both the numerator and denominator, one can always cancel common factors.

Computing large factorials though is difficult and time consuming, even with a calculator. Fortunately we can let drscheme do the hard work for us.

```
(define (factorial n)
     (if (= n 0)
          1
          (* n (factorial (- n 1)))))



;; Let's now evaluate 6!

(factorial 6)
=>  720
```

Question?

How many different ways can one shuffle a standard deck of 52 playing cards?

Answer

By the *multiplication principle* there are 52 choices for the first card, 51 for the second, 50 for the third, . . .

$$\begin{aligned}\text{\# of shuffles} &= 52 \cdot 51 \cdot 50 \cdot \cdots \cdot 3 \cdot 2 \cdot 1 \\ &\triangleq 52!\end{aligned}$$

Outrageous Claim: This number is so large that the order of the cards in every well shuffled deck has almost certainly never been realized before in the history of the universe!

Answer (cont.)

$$\begin{aligned}\text{\# of shuffles} &= 52! \\ &= 80,658,175,170,943,878,571,660,636,856,403,766, \\ &\qquad 975,289,505,440,883,277,824,000,000,000,000 \\ &\approx 80 \times 10^{66}\end{aligned}$$

This is a **HUGE** number.

Some other huge numbers

- Number of seconds in a century $\approx 4.5 \times 10^9$.
- Number of human beings alive on Earth $\approx 6.7 \times 10^9$.
- Number of Oreo cookies sold since 1912 $\approx 4.9 \times 10^{11}$.
- Federal debt (est. in US$ on 9/10/07) $\approx 9.0 \times 10^{12}$.
- Number of seconds that have elapsed since the Big Bang $\approx 4 \times 10^{17}$.
- Number of distinct positions in a $3 \times 3 \times 3$ Rubik's cube $\approx 4 \times 10^{19}$.
- Number of grains of sand on the earth $\approx 10^{21}$.
- Number of stars in the visible universe (Simon Driver, 2003) $\approx 7 \times 10^{22}$.
- Number of atoms in a human body $\approx 10^{28}$.
- Mass of the sun (in kilograms) $\approx 10^{30}$.
- Number of legal chess positions $\approx 10^{40}$.
- Number of distinct positions in a $4 \times 4 \times 4$ Rubik's Revenge $\approx 10^{45}$.
- Number of permutations of 52 playing cards $\approx 8 \times 10^{67}$.
- Number of distinct positions in a $5 \times 5 \times 5$ Rubik's Ultimate Cube $\approx 10^{74}$.
- Number of physical particles in the universe (inflationary model) $\approx 10^{80}$.
- Number of distinguishable games of go $\approx 10^{768}$