Problems in Greek Mathematics

1. Give a proof that $\sqrt{2}$ is irrational.

2. Sketch a proof that the angles in a triangle sum to 180 degrees. You can assume common-sense facts about parallel lines and alternate exterior/interior angles.

3. State and sketch a proof of Thales' Theorem. You can assume the result of #2 and that the base angles in an isosceles triangle are equal.

4. State and sketch a proof of the Pythagorean Theorem.

5. Hippocrates of Chios achieved the "quadrature of lunes" and showed that doubling the cube was equivalent to finding something called a "double mean proportion." In your own words, state what each of these amounts to.

6. Hippias of Elis used the "trisectrix" to trisect an angle. What is the trisectrix (i.e., how is it defined)? How is it used to trisect an angle? Why doesn't it really solve the problem?

7. Describe the three-dimensional construction which Archytas of Tarentum used to double the cube.

8. Describe the paradoxes of Zeno, and explain how they "prove" the impossibility of motion.

9. Find the numbers that are, respectively, the arithmetic, geometric, and harmonic means of 4 and 9.

10. Explain how, using only straightedge and compass, you would: draw a line through a point, parallel to a given line; bisect an angle; construct a right angle; find the center of a given circle; divide a line segment into n equal pieces; square a rectangle; double a square.

11. In class I showed W. Knorr's proposed reconstruction of Theodorus of Cyrene's method for showing the irrationality of certain square roots. Give proofs, in the "Knorr/Theodorus" style, of the irrationality of 7 and 13.