I write to fill what seems to be a slight gap in the proof of Theorem 1.

It does not seem clear that your polynomial \( f_{20} \) does not have repeated roots. There is no reason given why \( \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 \) is impossible. If you had defined your sextic resolvents as, say,

\[
\phi_1 = x_1^3 x_2 x_3 + \text{corresponding terms, etc.}
\]

then if \( f(x) = x^5 - 2 \), say, one finds that indeed \( \phi_2 = \phi_3 = \phi_4 = \phi_5 = \phi_6 = 0 \), so the question is not an idle one. If the discriminant of \( f_{20} \) were a power of that of \( f \) then clearly \( f_{20} \) could not have repeated roots but as you point out, this is not the case.

However one can argue as follows. If \( f_{20} \) has repeated roots then by the Galois action, \( \theta_2 = \theta_3 \ldots = \theta_6 \). Now

\[
\theta_2 - \theta_3 = (x_1 - x_4)(x_3 - x_5)[(x_2 - x_1)(x_2 - x_4) - (x_2 - x_3)(x_2 - x_5)]
\]

and we deduce that 

\[
(x_2 - x_1)(x_2 - x_4) = (x_2 - x_3)(x_2 - x_5).
\]

Similarly, as \( \theta_4 = \theta_5 \), then

\[
\theta_4 - \theta_5 = (x_3 - x_4)(x_1 - x_5)[(x_2 - x_3)(x_2 - x_4) - (x_2 - x_1)(x_2 - x_5)]
\]

implies

\[
(x_2 - x_3)(x_2 - x_4) = (x_2 - x_1)(x_2 - x_5).
\]

As the differences \( x_i - x_j \) are non zero one gets

\[
(x_2 - x_1) = \pm(x_2 - x_3) \quad \text{and} \quad (x_2 - x_4) = \pm(x_2 - x_5),
\]

where the two \( \pm \) signs are the same. But \( x_2 - x_1 = (x_2 - x_3) \) implies that \( x_1 = x_3 \), which is not possible, so the \( - \) signs are correct, and

\[
2x_2 = x_1 + x_3 \quad \text{and} \quad 2x_2 = x_4 + x_5.
\]

But then \( 5x_2 = x_1 + x_2 + x_3 + x_4 + x_5 \) is rational, giving a contradiction.